Playing with probabilities in selective broadcast networks

Cassting 2015

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joint work with Nathalie Bertrand and Arnaud Sangnier

Motivation

- Distributed algorithms (*mutual exclusion, leader election, ...*)
- Telecommunication protocols (*routing, ...*)
- Algorithms for *ad-hoc* networks
- Model for biological systems

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All participants have the same behavior

They form a **crowd** [Esparza, STACS'14]

Crowd networks

- Every process follows a same given protocol
- Processes can communicate, by either
  - Message passing
  - Shared variables
  - *Rendez-vous* communications
  - Broadcast communications
  - **Multi-diffusion (selective broadcasts)**

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Parameterized verification of crowd networks

Does the network conform to a given specification independently of the number of participants?

In this talk

Decidability and complexity
of parameterized reachability problems
in probabilistic networks

Features:

- **Probabilistic protocols**
- **Multi-diffusion communications**
- **Simple reachability questions**

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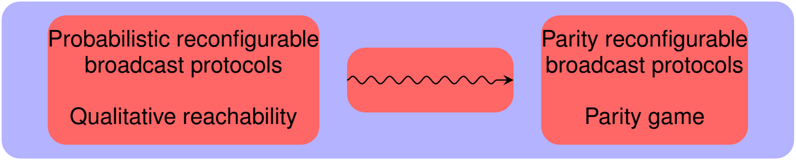
- **Multi-diffusion communications**
- **Simple reachability questions**

**Challenge:
parameterized system + non-determinism + probabilities**

Outline

Parameterized verification of probabilistic reconfigurable networks



Section 1 Section 3 Section 2

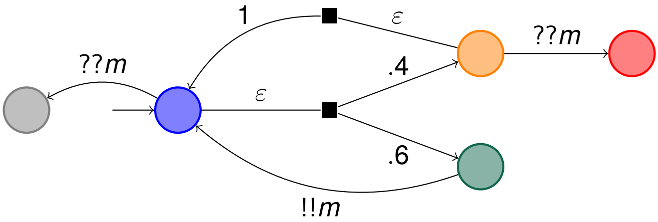
Outline

1. Probabilistic reconfigurable broadcast networks

2. Parity reconfigurable broadcast networks

3. Solving probabilistic networks via parity networks

A model for probabilistic protocols



Probabilistic protocol

Finite state system whose transitions are labelled with:

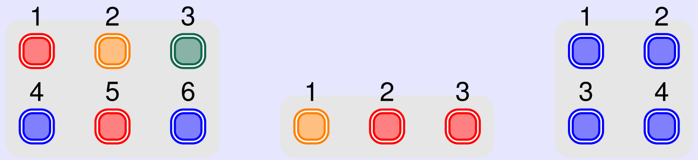
1. probabilistic internal actions - ε
2. broadcast of messages - $!!m$
3. reception of messages - $??m$

for m in a finite alphabet Σ.

A probabilistic protocol defines a probabilistic network

Configurations

Configurations: vectors of arbitrary size



- **Initial configurations**: **all** nodes are in the initial state

Remarks:

- Size of configurations is not bounded

$\Rightarrow$ **Networks are infinite state systems**

Probabilistic Networks: semantics

Markov decision process over set of configurations.

- C: (infinite) set of configurations
- $\Rightarrow$: $C \times C \cup C \times Dist(C)$: Transition relation
- C_0: (infinite) set of initial configurations

The number of nodes does not change along an execution

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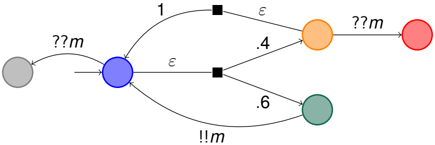
Transition relation

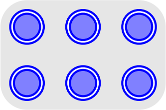
Decomposed in three steps

1. Choice of a process
2. Choice of a reception set (= set of neighbours)
3. Execution of an action
   - **local action** - the process performs an internal action ε
   - **communication** - the process sends a message ($!!m$), and its neighbours receive it ($??m$) if they can

Schedulers to resolve non-determinism

Scheduler π resolves the non-determinism by choosing the active process, its action and its neighbours





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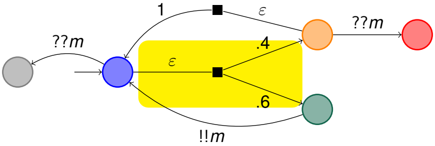

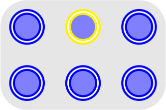

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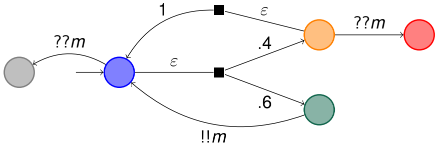

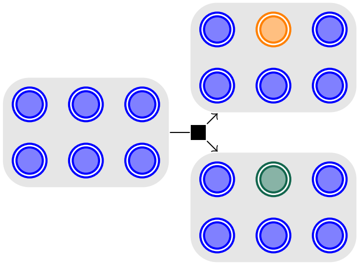

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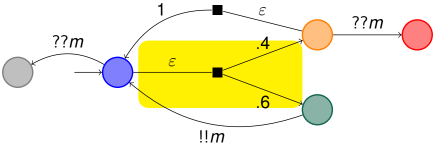

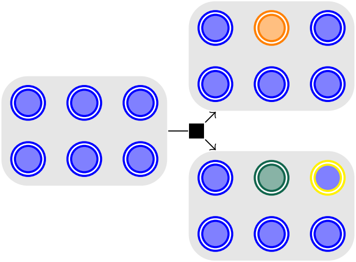

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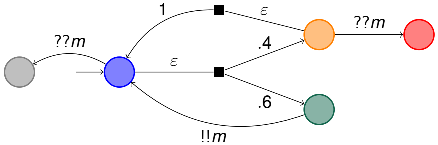

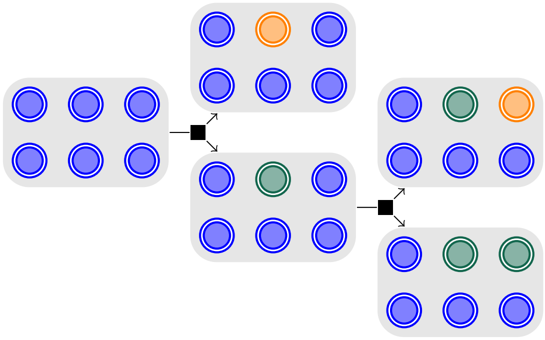

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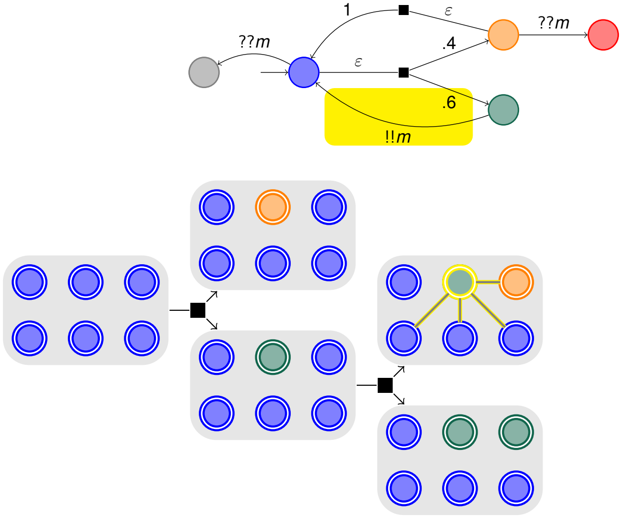

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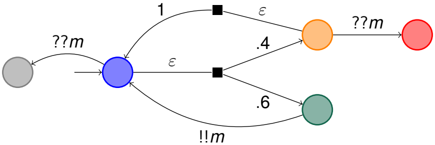

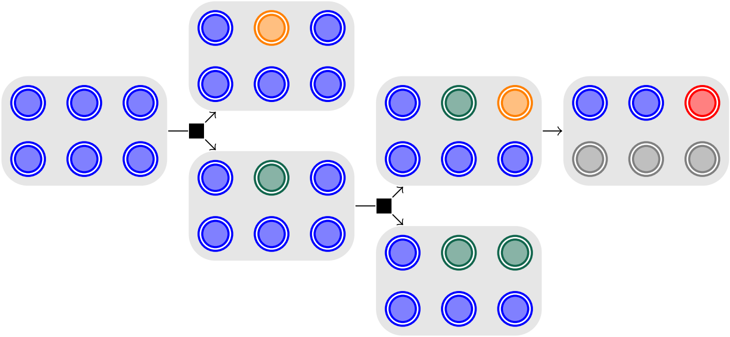

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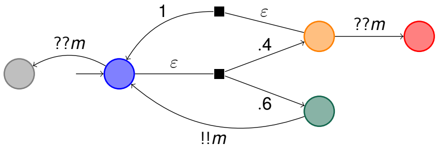

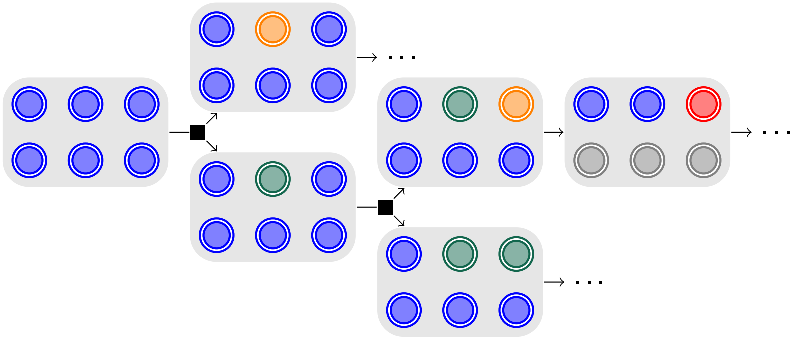

Parameterized reachability problems

scheduler π on N nodes induce a finite Markov chain of measure $\mathbb{P}_\pi^N$

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- Is an error state almost surely reachable, under some scheduler, and for some number of nodes?

$$\exists N,\ \exists \pi,\ \mathbb{P}_\pi^N(\Diamond q_{\mathtt{err}}) = 1$$

- Is an error state avoidable almost surely, under all adversarial schedulers, and for any number of nodes?

$$\forall N,\ \forall \pi,\ \mathbb{P}_\pi^N(\Diamond q_{\mathtt{err}}) = 0$$

Parameterized reachability problems

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$\text{Reach}_{opt}^{\sim b}$ $\quad opt \in \{\min, \max\}, \sim \in \{>, <, \leq, \geq, =\}, b \in \{0, 1\}$

Input: A process and a control state $q_F \in Q$;

Output: Does there exists N such that $\underset{\pi}{opt}\left\{\mathbb{P}_\pi^N(\Diamond q_F)\right\} \sim b$?

Monotocity property and consequences

Monotonicity

With more nodes in the network, the maximum reachability probability can only increase.

Idea: ignore additional nodes

As a consequence, *e.g.*

$$\exists N,\ \exists \pi,\ \mathbb{P}^N_\pi(\Diamond q_F) = 0 \Longleftrightarrow \exists \pi,\ \mathbb{P}^1_\pi(\Diamond q_F) = 0$$

$\textsc{Reach}^{=0}_{\max}$ is decidable in $\textsc{PTime}$ by considering a single node.

Solving $\text{REACH}_{\text{max}}^{>0}$

Does there exists a N and a scheduler π such that $\mathbb{P}_{\pi}^{N}(\Diamond q_F) > 0$?

- Equivalent to parameterized control state reachability
- **Decidable in PTIME** [Delzanno et al., FSTTCS'12]
- One can compute the set of reachable control states in PTIME

Solving $\textsc{Reach}^{>0}_{\max}$

Does there exists a N and a scheduler π such that $\mathbb{P}^N_\pi(\Diamond q_F) > 0$?

- Equivalent to parameterized control state reachability
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Not as easy for $\textsc{Reach}^{=1}_{\max}$!

Finite vs infinite MDPs

- Qualitative reachability is solvable in PTIME for finite MDPs by simple graph algorithms.
- Qualitative reachability in infinite-state MDPs: restricted to particular classes with *ad hoc* algorithms
  - non-deterministic and Probabilistic Lossy Channel Systems [Baier et al. 2007]
  - recursive Markov Decision Processes [Etessami et al. 2015]

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How to adapt this methodology to probabilistic networks?

Main issues:

1. Transform MDP into equivalent parity game **at the protocol level**
2. Solve parity networks

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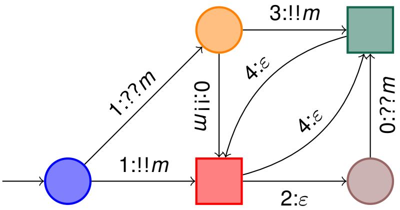
A model for parity protocol

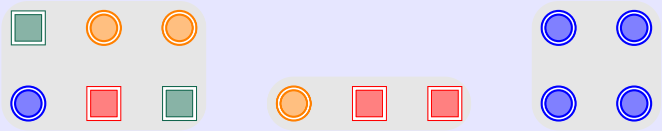


Parity protocol

-  states of Player 1
-  states of Player 2
- Transitions are labelled with:
  1. internal actions from Player 2's states – ε
  2. broadcast of messages – !!m
  3. reception of messages – ??m
  4. parities in $\mathbb{N}$

Semantics

Configurations: vectors of arbitrary size



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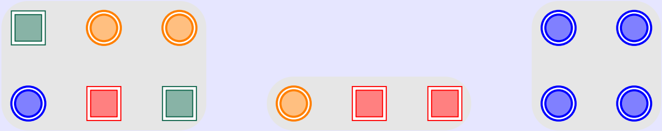


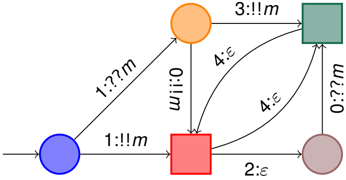
Roles are asymmetric

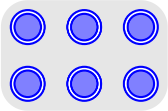
- Player 1 chooses the active process, and its neighbours
- If the active process is in a Player i's state, Player i chooses its action

Strategy profile (σ, τ) yields a play ρ

In communication transitions, the parity is the one of the corresponding broadcast

An example of play





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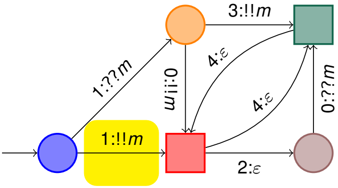

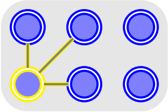

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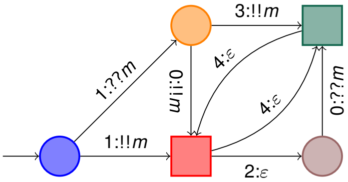

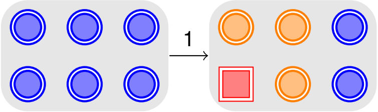

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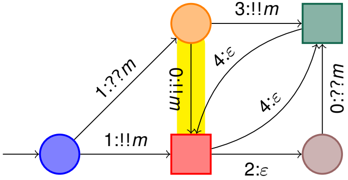

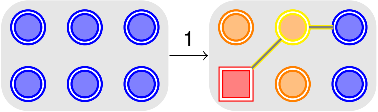

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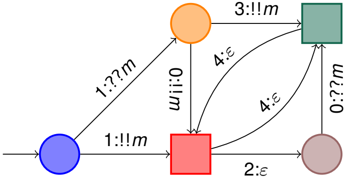

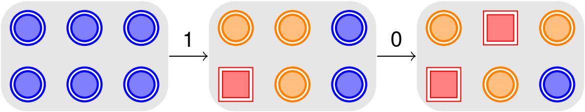

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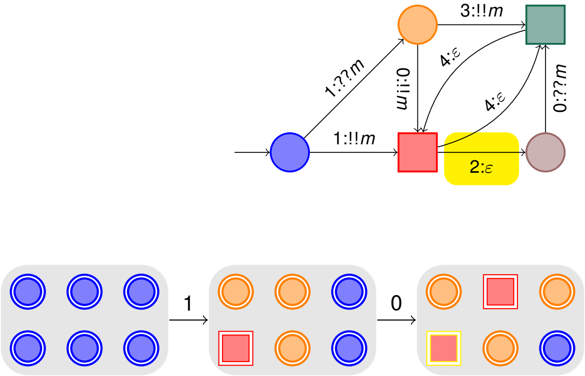

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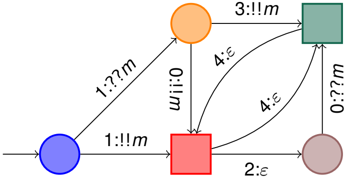

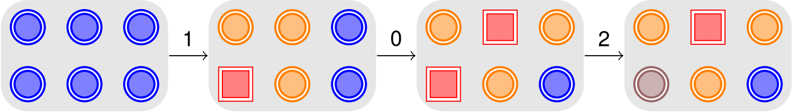

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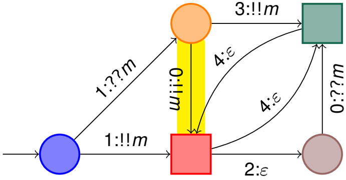

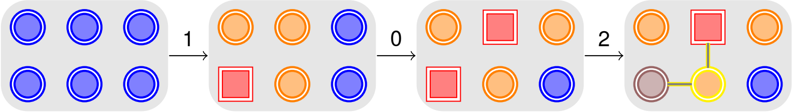

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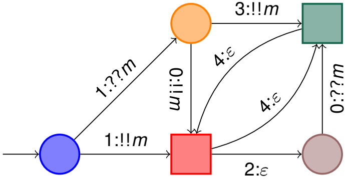

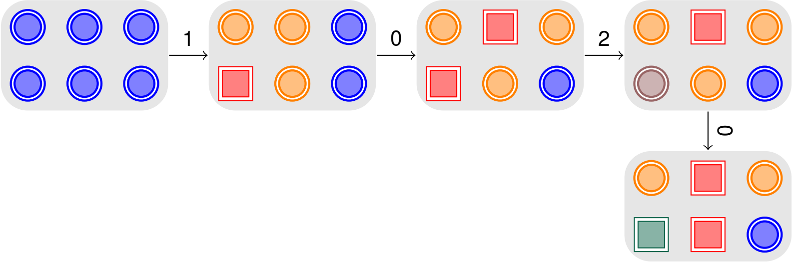

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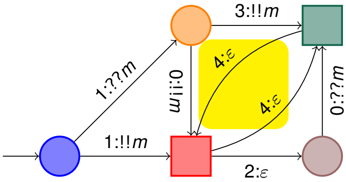

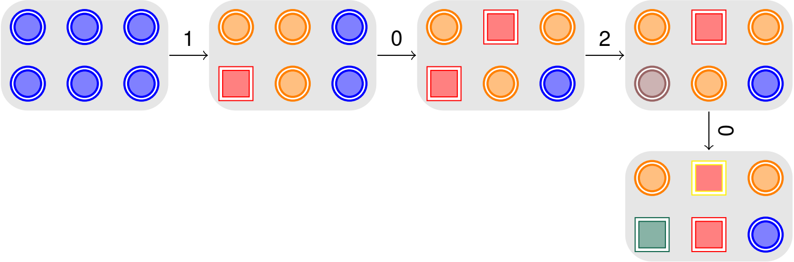

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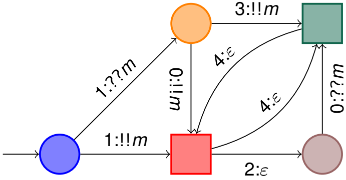

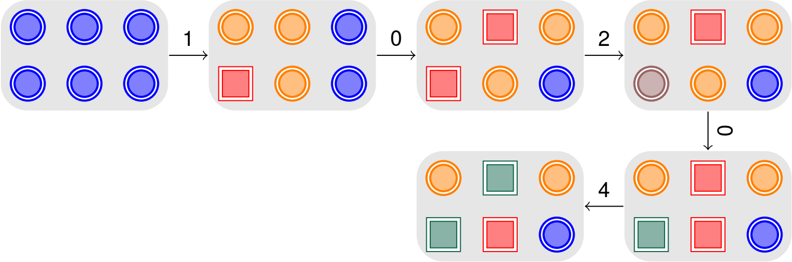

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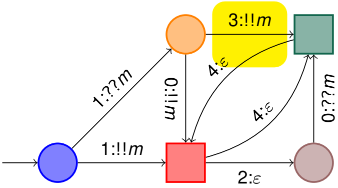

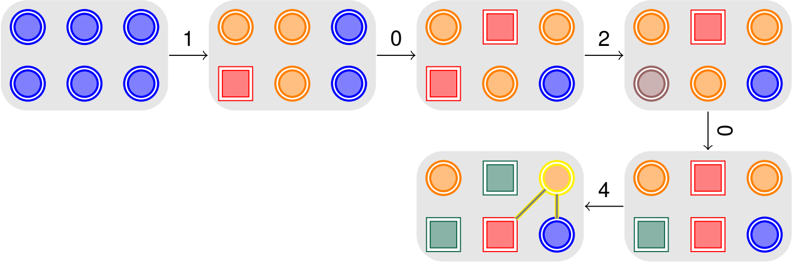

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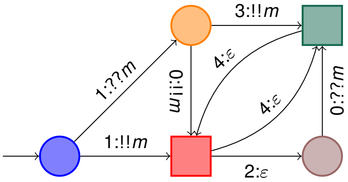

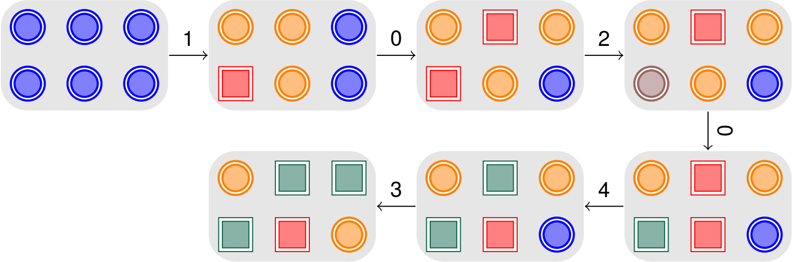

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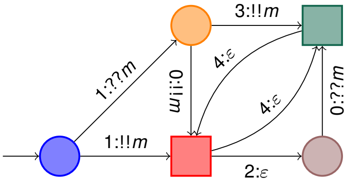

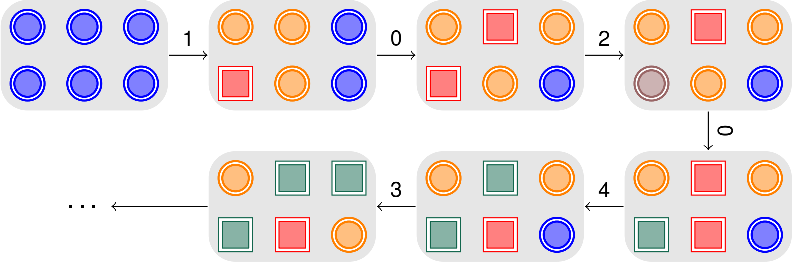

Parameterized game problem

Winning condition

$Win = \{\rho \texttt{ infinite play} \mid \max\{c | \#_c(\rho) = \infty\} \texttt{ is even}\}$

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Does Player 1 has a winning strategy for the parity objective for some number of nodes?

Game problem for parity networks

Input: A parity protocol P
Question: Does there exists N and a strategy σ for Player 1 such that for all strategies τ for Player 2 $\rho(\sigma, \tau, N) \in Win$.

Solving games on parity networks

Two steps

1. state-based strategies for Player 2 are enough
2. decidability of the existence of an infinite cycle in reconfigurable broadcast networks (*i.e.* networks of 1-player games)

State-based strategies for Player 2

- only depend on the control state labeling the active node
- there are finitely many
- given a fixed state-based strategy for Player 2, one obtains a reconfigurable broadcast network

Step 1: Restricting to state-based strategies

Proposition

Player 1 has a winning strategy against any **state-based** strategy of Player 2

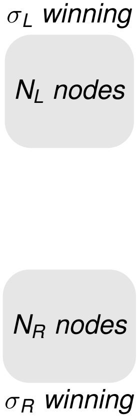
$$\Leftrightarrow$$

Player 1 has a winning strategy against any strategy of Player 2

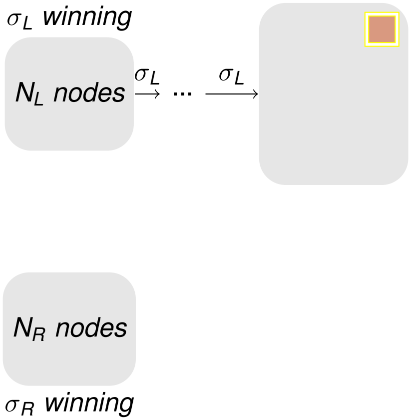
Proof by induction of the number of states of Player 2

- For the induction step, isolate a Player 2 state with two possible internal actions ε_L and ε_R
- By induction, if edge ε_R is deleted, Player 1 has a winning strategy σ_L for N_L nodes, and symmetrically
- A winning strategy is obtained combining σ_L and σ_R on $N_L + N_R$ nodes

Building a strategy using σ_L and σ_R



Building a strategy using σ_L and σ_R



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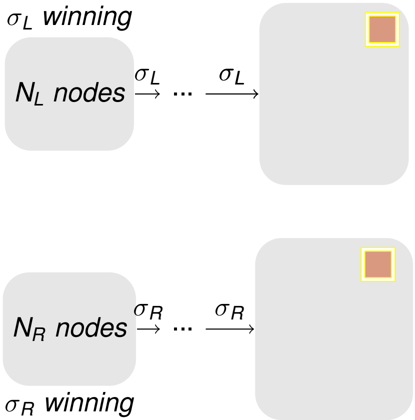

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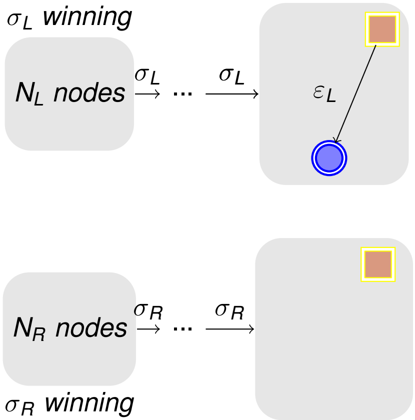

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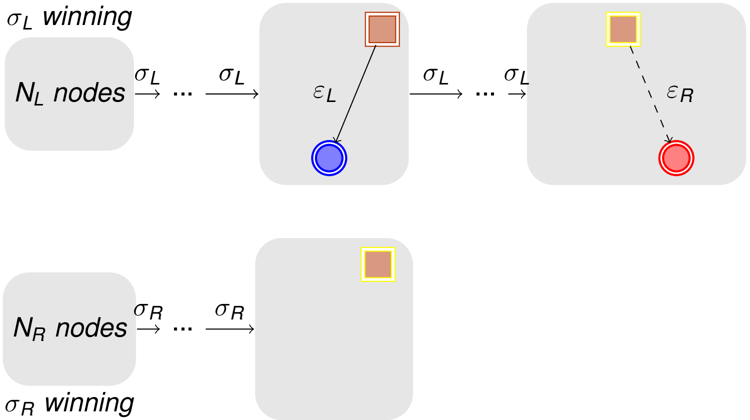

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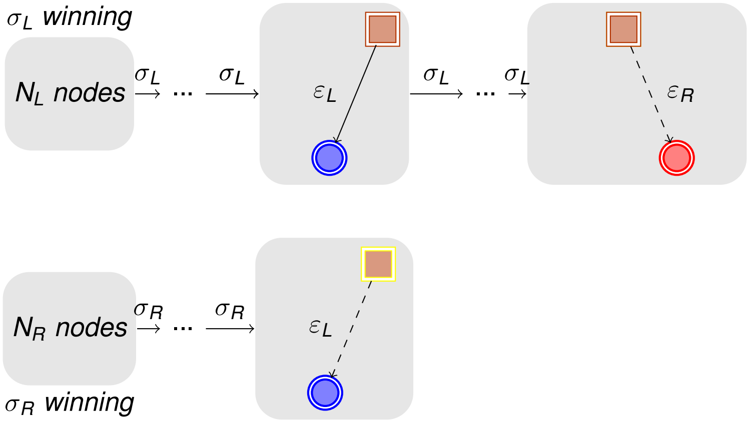
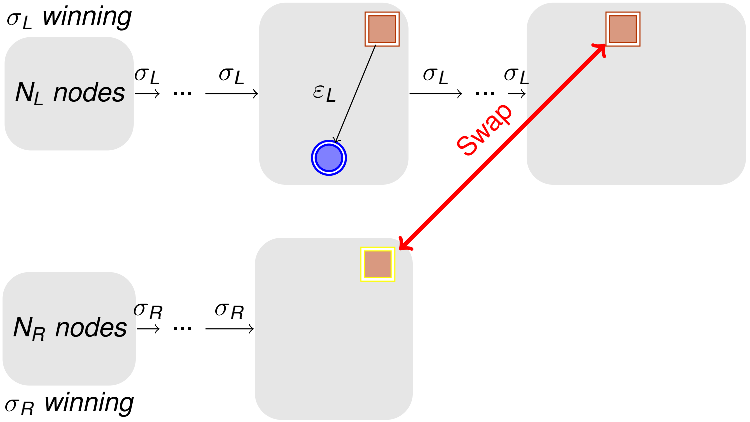
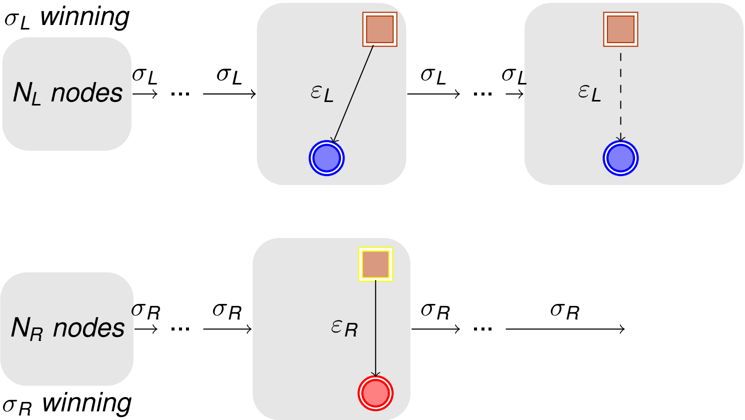


Building a strategy using σ_L and σ_R

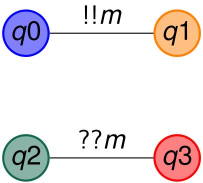


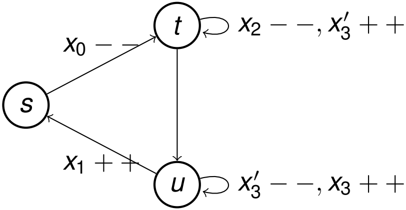
Building a strategy using σ_L and σ_R



Step 2: Detecting infinite paths

- For a fixed state-based strategy for Player 2, one obtains a reconfigurable broadcast network
- An infinite path corresponds to a positive cycle in a VASS





[Kosaraju & Sullivan 1988]

Detecting positive cycles in VASS can be done in PTIME

Deciding the game problem for parity networks

Theorem

The game problem for parity RBN is in coNP.

Proof idea:

- Guess a state-based strategy τ for Player 2
- Check whether it is winning for any number of nodes and against any strategies for Player 1
  - If the VASS has a positive cycle, τ it is not winning
  - Can be decided in PTIME
- If the state-based strategy τ is winning, then return NO

Outline

1. Probabilistic reconfigurable broadcast networks
2. Parity reconfigurable broadcast networks
3. **Solving probabilistic networks via parity networks**

Solving $\textsc{Reach}^{=1}_{\max}$: $\exists N,\ \exists \pi,\ \mathbb{P}^{N}_{\pi}(\Diamond q_F) = 1$

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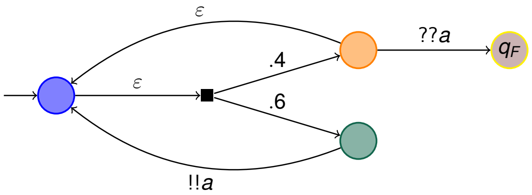
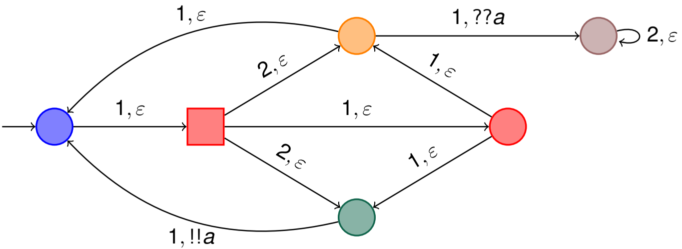


Idea of the reduction:

- Player 2 decides the outcome of probabilistic choices
- Fairness is ensured using parities



Correctness of the reduction for $\text{REACH}^{=1}_{\max}$

configurations in prob. network $\equiv$ configurations in parity network

schedulers $\equiv$ Player 1 strategies

Correctness of the reduction for $\textsc{Reach}_{max}^{=1}$

configurations in prob. network $\equiv$ configurations in parity network
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Key: $\textsc{Reach}_{max}^{=1}$ iff from every reachable configuration there is a path to a target configuration

Correctness of the reduction for $\textsc{Reach}^{=1}_{\max}$

configurations in prob. network $\equiv$ configurations in parity network
schedulers $\equiv$ Player 1 strategies

Key: $\textsc{Reach}^{=1}_{\max}$ iff from every reachable configuration there is a path to a target configuration

Proof idea:

- If Player 1 has a winning strategy
  - **case 1** Player 2 always decides the outcome of probabilistic choices; corresponds to paths in null measure set
  - **case 2** Player 2 eventually always leave decision to Player 1; from each reachable configuration, there is a path to the target
- If Player 1 has no winning strategy
  For every σ, Player 2 eventually lets Player 1 decide the outcome of probabilistic choices;
  there exists a configuration from which target is not reachable

Complexity of almost-sure reachability

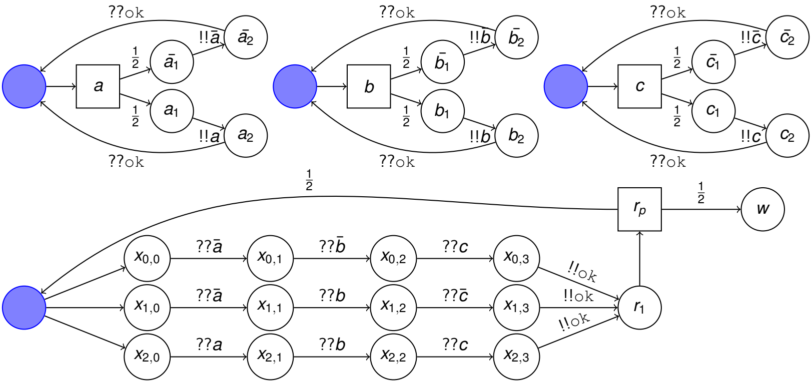
Theorem

$\textsc{Reach}^{=1}_{\text{max}}$ is $\textsc{co}$NP-complete.

- membership in $\textsc{co}$NP by reduction to games on parity networks
- $\textsc{co}$NP-hardness is obtained by reducing UNSAT

coNP-hardness of almost-sure reachability

$$\varphi = (a \vee b \vee \bar{c}) \wedge (a \vee \bar{b} \vee c) \wedge (\bar{a} \vee \bar{b} \vee \bar{c})$$



If φ is UNSAT, for any assignment, choose a clause so that the probability to reach w is .5.

Conclusion

Summary

- **model**: probabilistic selective broadcast networks
- **properties**: parameterized qualitative reachability questions
- **resolution**: via parity networks, yet another new model
- **complexities**: PTIME or coNP-complete

Conclusion

Summary

- **model**: probabilistic selective broadcast networks
- **properties**: parameterized qualitative reachability questions
- **resolution**: via parity networks, yet another new model
- **complexities**: PTIME or CONP-complete

Perspectives

- move to quantitative questions
- beyond reachability
- consider other communication means
- schedulers taking into account processes local view