Classifying Recognizable Infinitary Trace Languages Using Word Automata

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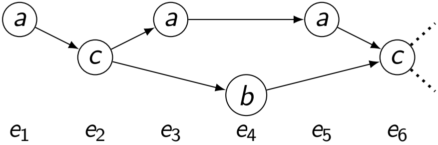
RWTH AACHEN UNIVERSITY

4th Cassting Meeting, Aachen
October 30, 2014

traces and trace languages

Mazurkiewicz traces model concurrent executions of distributed systems. . .



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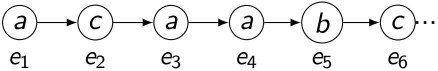

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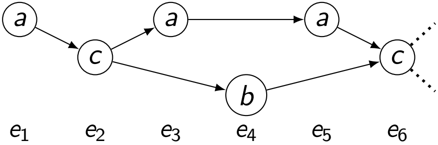

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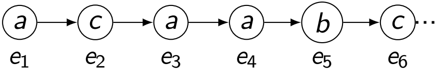
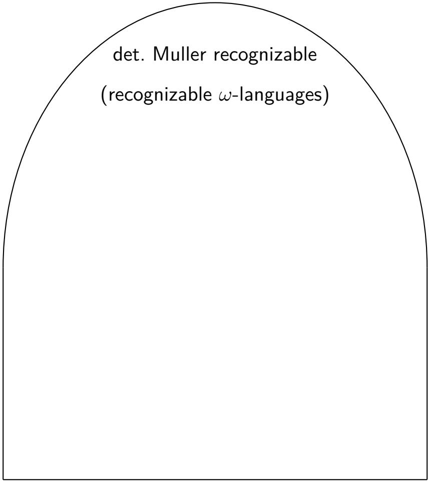


Objective: Generalize the key results of ω-word languages to the case of ω-trace languages – a problem open since the 1990's.

the Borel classification of recognizable ω-languages



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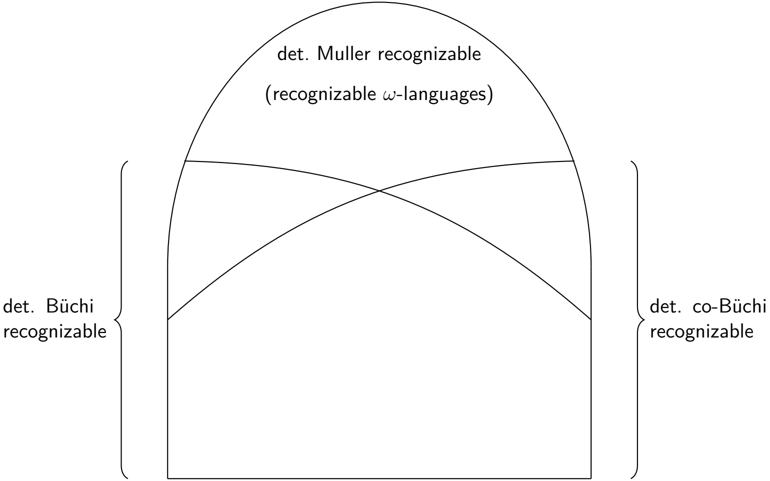

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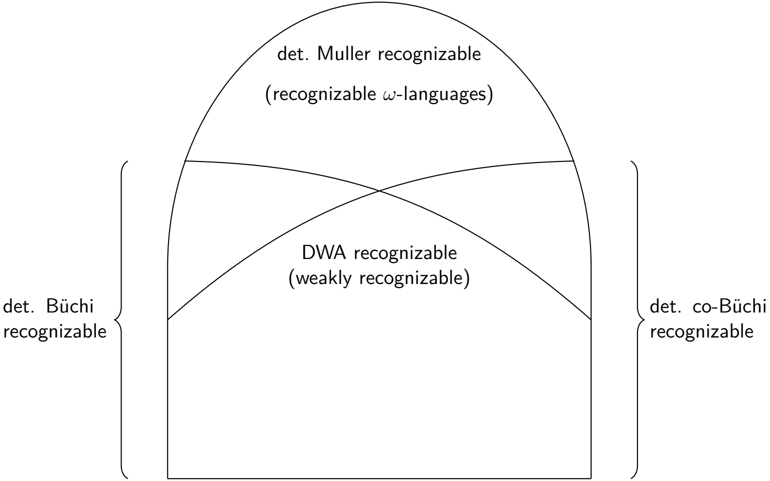

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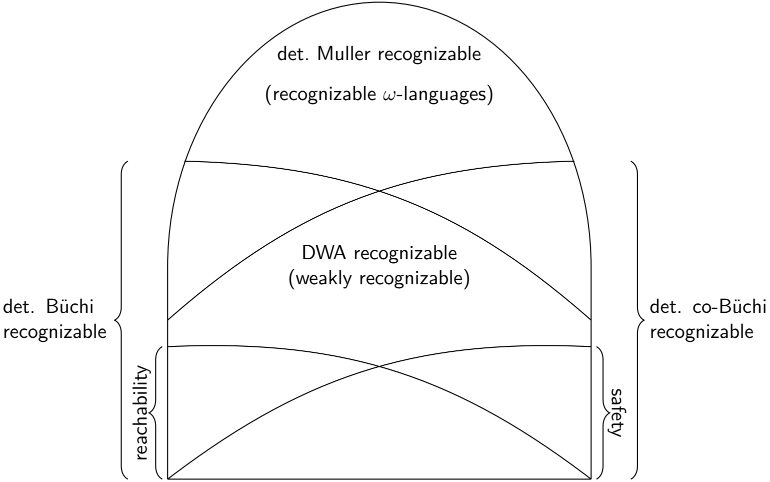

motivation

Why classify recognizable ω-languages?

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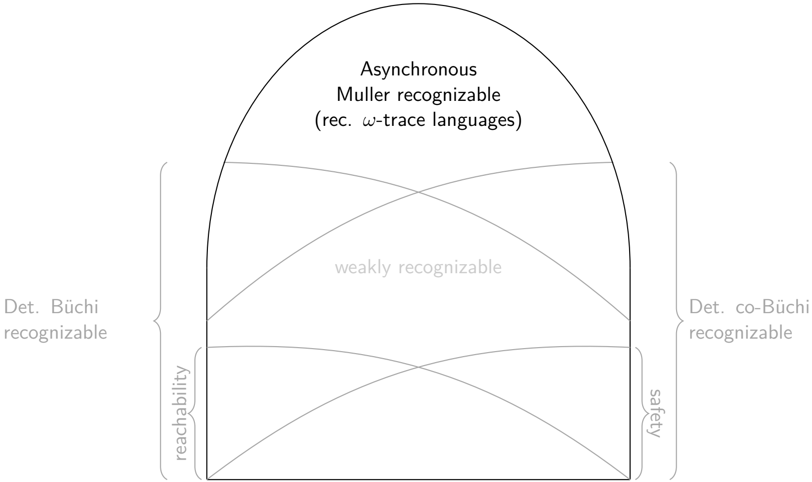
Why classify recognizable ω-languages?

1. Natural descriptions of recognizable ω-languages with the help of regular languages (using reachability, safety, and liveness conditions)
2. Easy constructions of ω-automata from DFAs
3. Decidability of the level of a given recognizable ω-language
4. Efficient algorithms for synthesis and verification for different classes

the case of recognizable ω-trace languages

[Diekert & Muscholl, 1993]



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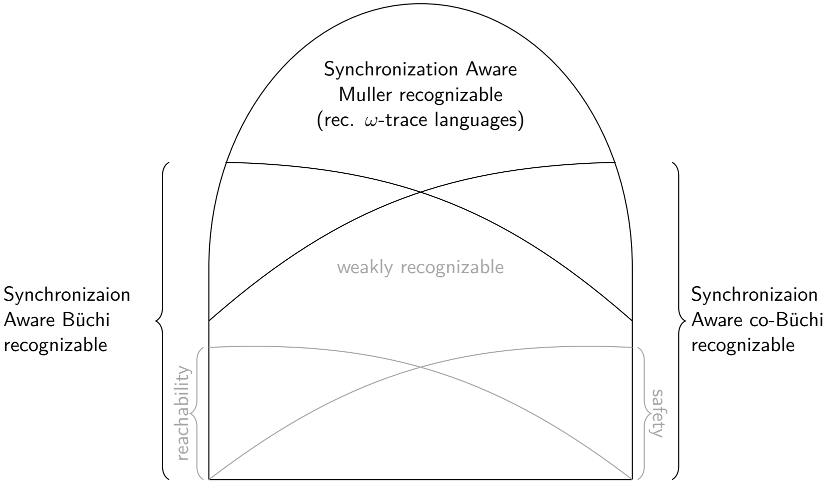
[C., 2014]



a new classification of trace languages

Invoking the framework of **trace-closed** languages:

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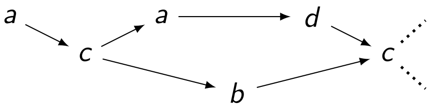

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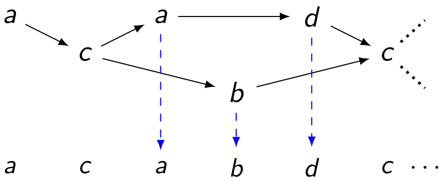

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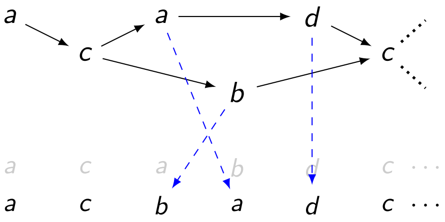

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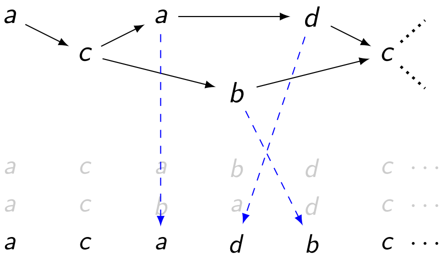

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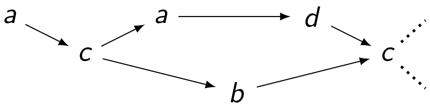

$a \quad c \quad a \quad b \quad d \quad c \cdots$

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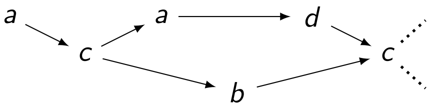

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Objective

Classification of recognizable ω-trace languages in terms of word automata recognizing trace-closed ω-languages.

preliminaries: traces

Let $I \subseteq \Sigma^2$ be a symmetric, irreflexive **independence relation**, then (Σ, I) is called an **independence alphabet**.

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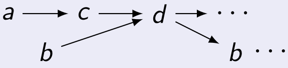
Definition (Finite and infinite traces)

A **trace** over (Σ, I) is a labeled DAG:



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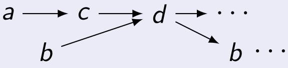
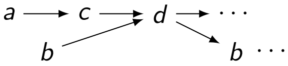


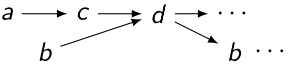
The set of all finite traces: $\mathbb{M}(\Sigma, I)$; the set of all infinite traces: $\mathbb{R}(\Sigma, I)$

preliminaries: trace-closed languages

- Mapping words to traces, $\Gamma\colon \Sigma^* \to \mathbb{M}(\Sigma, I)$ or $\Gamma\colon \Sigma^\omega \to \mathbb{R}(\Sigma, I)$

  $\Gamma(abcdb\ldots) =$ 

- Mapping traces to trace-closed sets of words

  $\Gamma^{-1}\Big($  $\Big) = \{bacdb\ldots, abcdb\ldots, acbdb\ldots\}$

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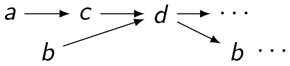
  $\Gamma(abcdb\ldots) =$ 

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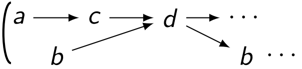
  $\Gamma^{-1}\Big($ 

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- Words u and v are **trace-equivalent**, $u \sim_I v$, if $\Gamma(u) = \Gamma(v)$
- A finite or infinite language L is **trace-closed** if $L = \Gamma^{-1}(\Gamma(L))$

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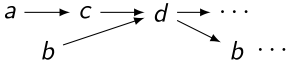
$\Gamma(abcdb\ldots) =$ 

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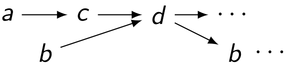
$\Gamma^{-1}\Big($  $\Big) = \{bacdb\ldots, abcdb\ldots, acbdb\ldots\}$

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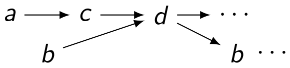
$\Gamma(abcdb\ldots) =$ 

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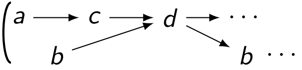
$\Gamma^{-1}\Big($  $\Big) = \{bacdb\ldots, abcdb\ldots, acbdb\ldots\}$

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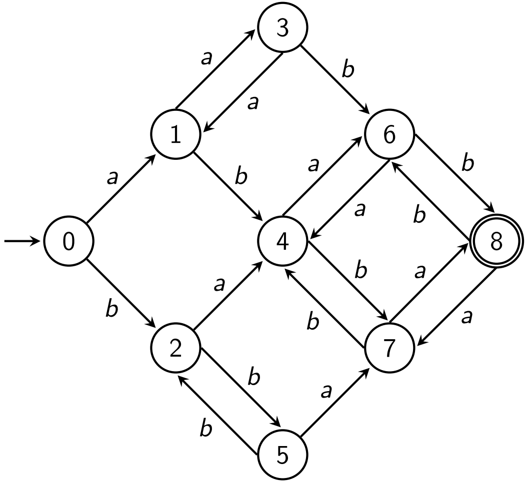
Theorem (Zielonka, 1987; resp. Gastin & Petit, 1991)

*A trace language $T \subseteq \mathbb{M}(\Sigma, I)$, resp. $\Theta \subseteq \mathbb{R}(\Sigma, I)$, is **recognizable** iff $\Gamma^{-1}(T)$, resp. $\Gamma^{-1}(\Theta)$, is a recognizable word language.*

preliminaries: I-diamond automata

Finitary trace-closed languages are recognized by ***I*-diamond** DFA.

Let $\Sigma = \{a, b\}$, aIb. Define $K := [(aa)^+(bb)^+]_{\sim_I}$.



a Borel-like hierarchy of trace-closed languages?

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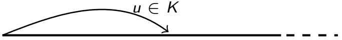
- $\text{ext}(K) := \{\alpha \in \Sigma^\omega \mid \alpha =$ 

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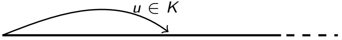
- $\mathrm{ext}(K) := \{\alpha \in \Sigma^\omega \mid \alpha =$ 

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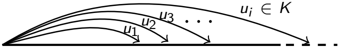
- $\lim(K) := \{\alpha \in \Sigma^\omega \mid \forall i \in \mathbb{N}, \alpha =$ 

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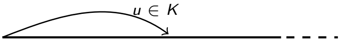
- $\mathrm{ext}(K) := \{\alpha \in \Sigma^\omega \mid \alpha =$ 

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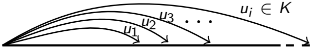
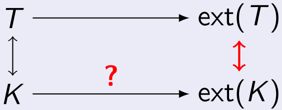
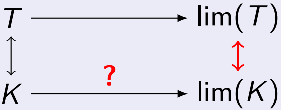
- $\lim(K) := \{\alpha \in \Sigma^\omega \mid \forall i \in \mathbb{N}, \alpha =$ 

$\}$

Describing recognizable ω-trace languages in terms of reachability and liveness of recognizable trace-closed languages:





reachability of finitary properties

Definition

For any T, and $K = \Gamma^{-1}(T)$, the **infinitary extension** of T is the ω-trace language $\text{ext}(T) := \Gamma(\text{ext}(K)) = T \cdot \mathbb{R}(\Sigma, I)$.

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- $\mathrm{ext}(K) \stackrel{?}{=} \Gamma^{-1}(\mathrm{ext}(T))$

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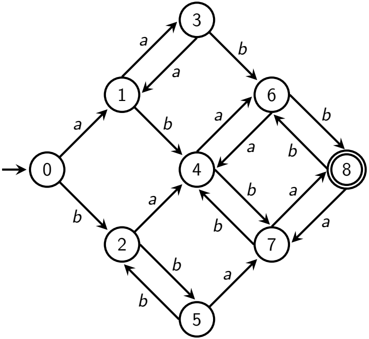
  $$\Gamma((aabb)^\omega) = \Gamma(ab(aabb)^\omega) = \begin{array}{c} a \longrightarrow a \longrightarrow a \longrightarrow \cdots \\ b \longrightarrow b \longrightarrow b \longrightarrow \cdots \end{array} \in \mathrm{ext}(T)$$

For trace-closed $K \in \mathrm{REC}$, $\mathrm{ext}(K)$ may not be trace-closed.

the I-suffix extension



- $K = [(aa)^+(bb)^+]_{\sim_I}$ with aIb

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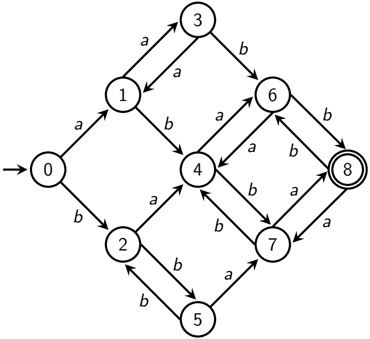

- $K = [(aa)^{+}(bb)^{+}]_{\sim_I}$ with aIb
  - $I_b = \{a\}$, $I_a = \{b\}$

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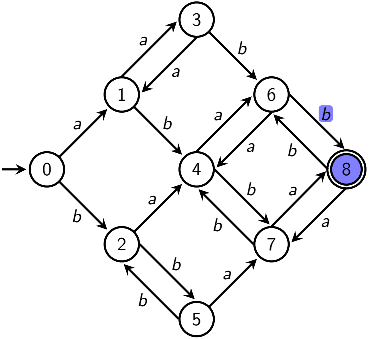

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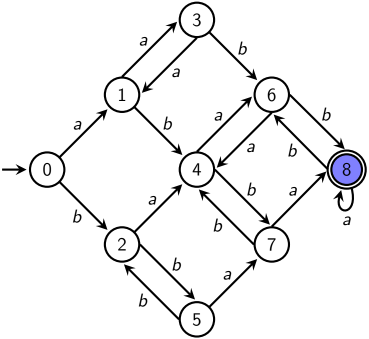

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- $Kb^{-1}b \cdot I_b^*$

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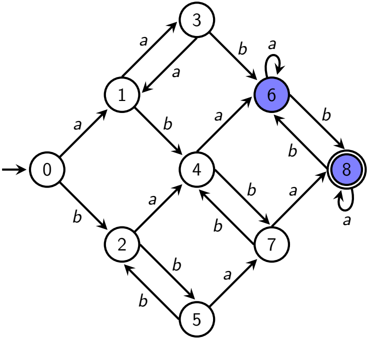

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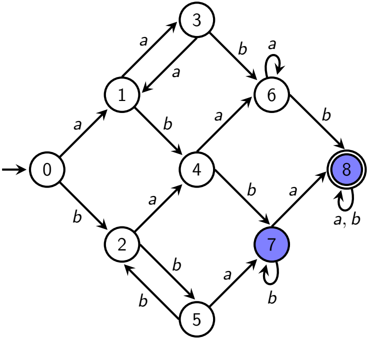

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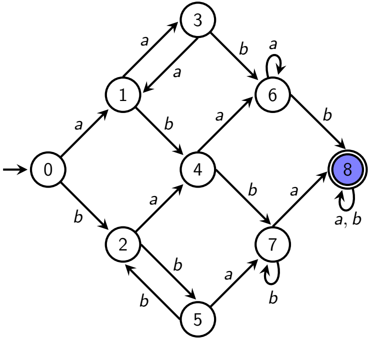
  - $I_b = \{a\}$, $I_a = \{b\}$
- $Kb^{-1}b$
- $Kb^{-1}b \cdot I_b^*$
- $K_{\{b\}} = K \cup [Kb^{-1}b \cdot I_b^*]_{\sim_I}$
- $K_{\{a,b\}} = K_{\{b\}} \cup [Ka^{-1}a \cdot I_a^*]_{\sim_I}$

the I-suffix extension



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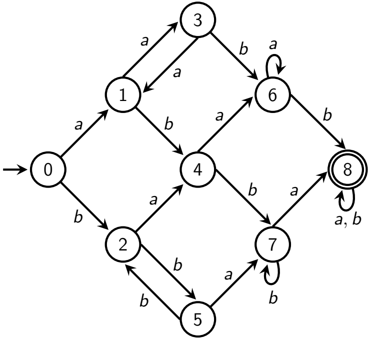
  - $I_b = \{a\}$, $I_a = \{b\}$
- $Kb^{-1}b$
- $Kb^{-1}b \cdot I_b^*$
- $K_{\{b\}} = K \cup [Kb^{-1}b \cdot I_b^*]_{\sim_I}$
- $K_{\{a,b\}} = K_{\{b\}} \cup [Ka^{-1}a \cdot I_a^*]_{\sim_I}$
- $K_I = K_\Sigma$

the I-suffix extension

Definition (I-suffix extension)

For $K \in \mathrm{REC}$, trace-closed, define $K_I := K \cup \bigcup_{a\in\Sigma}[Ka^{-1}a \cdot I_a^*]_{\sim_I}$



- $K = [(aa)^+(bb)^+]_{\sim_I}$ with aIb
  - $I_b = \{a\}$, $I_a = \{b\}$
- $Kb^{-1}b$
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weakly recognizable trace-closed languages

Lemma

For any trace language $T \in \mathrm{REC}$, *and* $K = \Gamma^{-1}(T)$, *it holds that* $\mathrm{ext}(K_I) = \Gamma^{-1}(\mathrm{ext}(T))$.

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Theorem (I-diamond DWA recognizability)

A trace-closed language $L \subseteq \Sigma^\omega$ is recognized by an I-diamond DWA iff $L \in \mathrm{BC}(\mathrm{ext}(\mathcal{K}))$ for a set $\mathcal{K} \subseteq \mathrm{REC}$ of trace-closed regular languages.

liveness of finitary properties

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Definition (Diekert & Muscholl, 1993)

For a trace language T, and $K = \Gamma^{-1}(T)$, the **infinitary limit** of T is defined as $\lim(T) := \Gamma(\lim(K))$.

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Let $\Sigma = \{a, b\}$, aIb, $K = [(aa)^+(bb)^+]_{\sim_I}$, and $T = \Gamma(K)$.

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  $aabbaabb \cdots \in \lim(K)$ and $abaabbaabb \ldots \notin \lim(K)$; but

  $$\Gamma((aabb)^{\omega}) = \Gamma(ab(aabb)^{\omega}) = \begin{array}{l} a \longrightarrow a \longrightarrow a \longrightarrow \cdots \\ b \longrightarrow b \longrightarrow b \longrightarrow \cdots \end{array} \in \lim(T)$$

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Proposition

For <u>no trace-closed language</u> $L \subseteq \mathrm{REC}$, i.h.t. $\lim(L) = \Gamma^{-1}(\lim(T))$.

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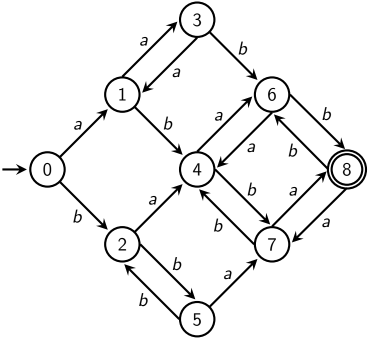
Definition (Limit-stability)

A trace-closed language $K \in \mathrm{REC}$ is **limit-stable** if $\lim(K)$ is trace-closed.

limit-stability and I-diamond automata

Definition (F, I-cycle closure)

For a given (Σ, I), an I-diamond DFA is F, I-**cycle closed** if for all $u \sim_I v$ and all $q \in Q$, $q \xrightarrow{u}_F q$ iff $q \xrightarrow{v}_F q$.



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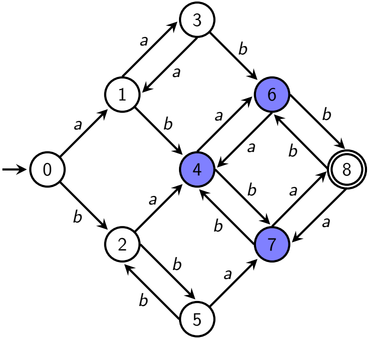

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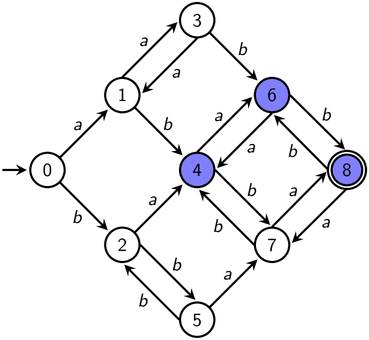

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F, I-cycle closure is necessary and sufficient

Theorem (Trace-closed infinitary limits)

A language $K \in \mathrm{REC}$ is limit-stable if and only if any DFA recognizing K is F, I-cycle closed.

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A language $K \in \mathrm{REC}$ is limit-stable if and only if any DFA recognizing K is F, I-cycle closed.
Which implies: An ω-language recognized by a DBA is trace-closed if and only if the DBA is F, I-cycle closed.

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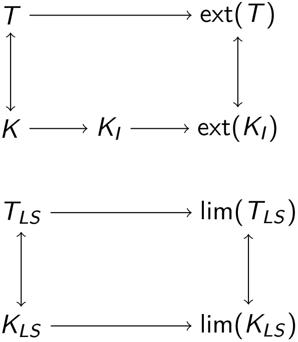
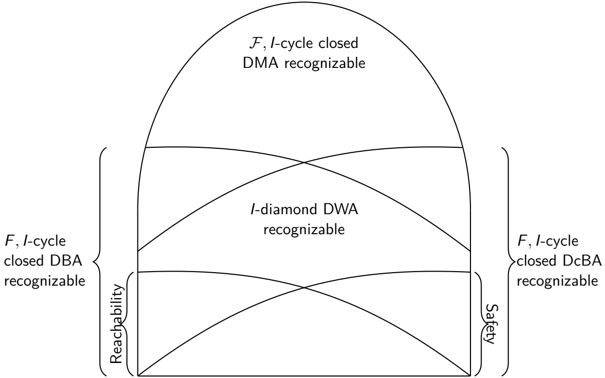
Theorem (Recognizable ω-trace languages)

A trace language Θ is recognizable if and only if $\Gamma^{-1}(\Theta)$ is a finite Boolean combination of F, I-cycle closed DBA recognizable languages.

a Borel-like hierarchy of recognizable ω-trace languages





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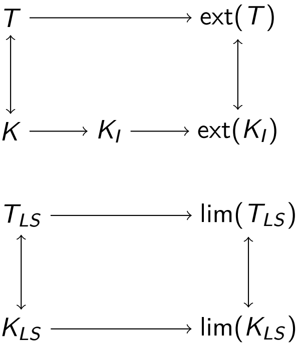

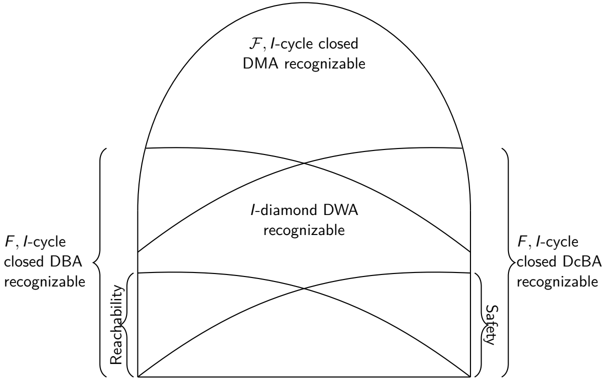

Still open

- Deciding F, I-cycle closed DBA recognizability à la Landweber
- Deciding distributed synthesis problem for specifications $\lim(K_{LS})$