



# Cassting

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## Solutions concepts for collective adaptive systems

Thomas Brihaye (UMONS),  
Nicolas Markey (CNRS)

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Thomas Brihaye, Nicolas Markey

## Abstract

This deliverable reports about our recent advances regarding interesting solutions concepts for collective adaptive systems. Compared to zero-sum games, where expressing one winning objective is enough, non-zero-sum games have much richer solution concepts, including (but not exclusively) equilibria. In the very same way as when defining various formalisms for modelling collective adaptive systems, we aim at defining various formalisms for specifying properties of collective adaptive systems, looking for a trade-off between expressiveness and tractability. This deliverable summarizes the contributions achieved by the Cassting consortium. The list of publications on page 11 lists the corresponding papers, which contain the full details.

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## Introduction

Game theory [OR94] is a branch of mathematics which has been successfully applied in various domains including economics, social sciences, biology, political sciences, engineering and computer science. Game theory attempts to model strategic situations where several individuals are interacting and try to predict what will be the decisions taken by individuals in a given situation, assuming rationality of those individuals. In this framework, a strategic situation is called a game and each individual taking part to this game is called a player, their decisions are called strategies. More recently a large current of work in game theory has arisen under the driving force of the computer-science community. This led to the new subfield of *algorithmic game theory* [NRTV07]. Nash theorem is a purely existential result, i.e., it ensures the existence of Nash equilibria but gives no clue on how we could compute them and how difficult this computation would be. The needs for computational and complexity information in this domain are well illustrated by a famous quotation from Kamal Jain (Microsoft Research), popularised by Christos Papadimitriou (Berkeley): “*If your laptop can’t find the equilibrium, then neither can the market.*” In the general case of non-zero-sum games, results concerning the complexity of finding Nash equilibria have only been obtained recently by Papadimitriou *et al* [DGP09].

The reason for the success of game theory in the field of automatic synthesis is that game-based models are well-suited to capture systems that are: *distributed*, where each player has his own objective, and where the decision making is *not centralised* and possibly *independent of the other players*; *collaborative*, where the global outcome of the game *emerges* from the interactions between the players; and *large scale*, where the multiple agents are modelled by the different players of the game (each with a potentially large sets of actions). *Game theory* is thus a first-class candidate for modelling collective adaptive systems and reasoning about them. Furthermore, adequate game models for collective adaptive systems would allow to apply automatic synthesis techniques to collective adaptive systems. However, game theory as it is applied nowadays in automatic synthesis requires *deep*

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- [OR94] Martin J. Osborne and Ariel Rubinstein. *A course in game theory*. MIT Press, 1994.
- [NRTV07] Noam Nisan, Tim Roughgarden, Eva Tardos, and Vijay V. Vazirani. *Algorithmic Game Theory*. Cambridge University Press, 2007.
- [DGP09] Constantinos Daskalakis, Paul W. Goldberg, and Christos H. Papadimitriou. The complexity of computing a Nash equilibrium. *SIAM Journal on Computing*, 39(1):195–259, 2009.

*extensions* in order to be able to cope with the unique conjunction of characteristics of collective adaptive systems (in particular their multi-scaleness, adaptiveness and openness).

During its first year of existence, the Cassting project already investigated several aspects of these questions, as we report in this deliverable:

- First step, we studied the recent concept of *secure equilibrium* which adapts the famous Nash equilibrium to the context of controller synthesis [CHJ06]. The results concerning secure equilibria are described in Section 1.
- In Section 2, we investigate new relevant notions of equilibria for collective adaptive systems. More precisely, we introduce the concepts of *doomsday equilibria* and of *k-fault tolerant Nash Equilibrium* and study algorithm to decide existence of such equilibria.
- While the notions of equilibria as above provide a good way of expressing properties of collective adaptive systems, they are not very fine-grained, and may fail to capture interesting properties to be verified on collective adaptive systems. Also, there will usually exist several equilibria for a given game, and requiring extra properties in order to identify *optimal* equilibria might reveal especially interesting. Logical formalisms (in particular extensions of temporal logics [AHK02]) are a powerful tool for this kind of problems. It is thus crucial to have a good understanding of the expressiveness and complexity of such logics. Advances in this direction are described in Section 3.

## 1 About secure equilibria

**Background.** Secure equilibrium [CHJ06] is a refinement of Nash equilibrium, which provides some security to the players against deviations when a player changes his strategy to another best response strategy. The concept of secure equilibrium is specifically developed for assume-guarantee synthesis

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[CHJ06] Krishnendu Chatterjee, Thomas A. Henzinger, and Marcin Jurdziński. Games with secure equilibria. *Theoretical Computer Science*, 365(1-2):67–82, November 2006.

[AHK02] Rajeev Alur, Thomas A. Henzinger, and Orna Kupferman. Alternating-time temporal logic. *Journal of the ACM*, 49(5):672–713, September 2002.

and has already been applied in this context [CR10]. Preliminary existence and algorithmic results on these concepts have been obtained in [CHJ06], only in the two-player qualitative case. Existence results for quantitative reachability games have been obtained in [De 13] in the two-player framework.

## 1.1 Secure equilibria in two-player games

**Our contributions** In [BMR14], we consider two-player non zero-sum infinite duration games played on weighted graphs. We prove that secure equilibria always exist in a large class of weighted games which includes common measures like sup, inf, lim sup, lim inf, mean-payoff, and discounted sum. Moreover we show that it is possible to synthesise such strategy profiles that are finite-memory and use few memory. Finally, we prove that the constrained existence problem for secure equilibria is decidable for sup, inf, lim sup, lim inf and mean-payoff measures. Our solutions rely on new results for zero-sum quantitative games with lexicographic objectives that are interesting on their own right.

**Participants:** Véronique Bruyère (UMONS)  
Noémie Meunier (UMONS)  
Jean-François Raskin (ULB)

**References:** [BMR14]

## 1.2 Secure equilibria in multiplayer games

**Our contributions** In [DPFK<sup>+</sup>14], we establish the existence of secure equilibrium in two classes of multi-player perfect information turn-based games: (1) in games with possibly probabilistic transitions, having countable state and finite action spaces and bounded and continuous payoff functions, and (2) in games with only deterministic transitions, having arbitrary

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- [CR10] Krishnendu Chatterjee and Vishwanath Raman. Assume-guarantee synthesis for digital contract signing. *CoRR*, abs/1004.2697, 2010.
- [CHJ06] Krishnendu Chatterjee, Thomas A. Henzinger, and Marcin Jurdziński. Games with secure equilibria. *Theoretical Computer Science*, 365(1-2):67–82, November 2006.
- [De 13] Julie De Pril. *Equilibria in Multiplayer Cost Games*. Ph.D. Thesis, University of Mons, Belgium, 2013.

state and action spaces and Borel payoff functions with a finite range (in particular, qualitative Borel payoff functions). To our knowledge, this is the first existence result concerning secure equilibria in multiplayer games. We show that these results apply to several types of games studied in the literature including qualitative games (with Borel winning conditions) and quantitative games such as quantitative reachability of discounted payoff.

Let us notice that the above results solve an open problem which was mentioned in the proposal of the Cassting project: “[...] *many open problems have to be solved, including the existence of secure equilibria in quantitative reachability games, [...]*”.

**Participants:** Julie De Pril (UMONS)

**References:** [\[DPFK<sup>+</sup>14\]](#)

## 2 New concepts of equilibria

**Background.** Once interesting models for collective adaptive systems are developed, another major challenge is to define appropriate solution concepts for expressing properties to be checked of those models. Classical concepts (such as Nash equilibria), while being relevant for several applications, are not always the most adequate solution concept for modelling the desired complex interactions of collective adaptive systems. That is why we defined new solution concepts.

A first differential for solution concepts is their interest regarding the desired behaviours of collective adaptive systems. Another differential for interesting solution concepts is their computational properties. Indeed, in the end we intend to automatically synthesise the behaviours of the systems, and we are thus looking for decidable solution concepts.

### 2.1 Doomsday equilibrium

**Our contributions.** Two-player games on graphs provide the theoretical framework for many important problems such as reactive synthesis. While the traditional study of two-player zero-sum games has been extended to multi-player games with several notions of equilibria, they are

decidable only for perfect-information games, whereas several applications require imperfect-information games.

In [CDFR14], we propose a new notion of equilibria, called *doomsday equilibria*, which is a strategy profile such that all players satisfy their own objective, and if any coalition of players deviates and violates even one of the players objective, then the objective of every player is violated.

We present algorithms and complexity results for deciding the existence of doomsday equilibria for various classes of  $\omega$ -regular objectives, both for imperfect-information games, and for perfect-information games. We provide optimal complexity bounds for imperfect-information games, and in most cases for perfect-information games.

**Participants:** Laurent Doyen (CNRS)  
Emmanuel Filiot (ULB)  
Jean-François Raskin (ULB)

**References:** [CDFR14]

## 2.2 Fault-tolerant equilibrium

**Our contributions** A major hypothesis in game theory is that the agents (or players) are *rational* entities. This hypothesis can be discussed when we work with computer systems. Indeed faulty behaviours can not be considered as rational. This motivates the introduction of a new concept of equilibrium that we named *k-fault tolerant Nash Equilibrium*. This concept is inspired from concepts such as *k-resilient equilibrium* [Aum59], and *k-immune equilibrium* [ADGH06] (which was also motivated by a potential irrationality of the agents).

The idea of *k-fault tolerant Nash Equilibrium* is that this is a Nash equilibrium such that no coalitions of at most  $k$  faulty players can bring

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[Aum59] Robert J. Aumann. Acceptable points in general cooperative  $n$ -person games. In *Contributions to the theory of games, Vol. IV*, Annals of Mathematics Studies, no. 40, pages 287–324. Princeton University Press, Princeton, N.J., 1959.

[ADGH06] Ittai Abraham, Danny Dolev, Rica Gonen, and Joseph Y. Halpern. Distributed computing meets game theory: robust mechanisms for rational secret sharing and multiparty computation. In *PODC*, pages 53–62. ACM, 2006.

the game into very bad issues. Our goal is to develop algorithms to find these equilibria in the spirit of the *suspect game* which has been intensively studied in [Bre12].

**Participants:** Patricia Bouyer (CNRS)  
Thomas Brihaye (UMONS)  
Quentin Hautem (UMONS)  
Nicolas Markey (CNRS)

### 3 Solution concepts expressed via temporal logics

**Background.** Temporal logics are a widely-used formalism in model checking: they are convenient for expressing properties to be checked of computerised systems. Temporal logic express properties on the order of occurrence of various events during the execution of the system. Formulas are built from atomic propositions (characterising the various states of the model), using temporal modalities such as "until". When used to express properties of branching structures, this is further extended with *path quantifiers*, to state that some or all executions satisfy a given subformula. The resulting logic is called CTL. When used on to express properties of games, they they can use *strategy quantifiers* in order to express the existence of strategies whose outcomes satisfy a given subformula. The resulting logic is called ATL [AHK02]. While the semantics of ATL given in [AHK02] is well-suited for zero-sum objectives, another semantics (henceforth called  $ATL_{sc}$  has been proposed in [BDLM09] for non-zero-sum objectives. The latter semantics can express various notions of equilibria, and much more.

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- [Bre12] Romain Brenguier. *Équilibres de Nash dans les Jeux Concurrents - Applications aux Jeux Temporisés*. Ph.D. Thesis, Laboratoire Spécification et Vérification, ENS Cachan, France, 2012.
- [AHK02] Rajeev Alur, Thomas A. Henzinger, and Orna Kupferman. Alternating-time temporal logic. *Journal of the ACM*, 49(5):672–713, September 2002.
- [BDLM09] Thomas Brihaye, Arnaud Da Costa, François Laroussinie, and Nicolas Markey. ATL with strategy contexts and bounded memory. In Sergei N. Artemov and Anil Nerode, editors, *Proceedings of the International Symposium Logical Foundations of Computer Science (LFCS'09)*, volume 5407 of *Lecture Notes in Computer Science*, pages 92–106. Springer-Verlag, January 2009.

**Our contributions** A model checking algorithm for  $ATL_{sc}$  was proposed a few years ago [DLM10], via tree automata. However, the logic was not fully understood, and several questions remained open, both regarding expressiveness of  $ATL_{sc}$  and regarding its algorithmic properties.

In a recent work, we noticed a very tight link between  $ATL_{sc}$  and another temporal logic, called QCTL (for Quantified CTL), introduced and partly studied long ago in various contexts. QCTL allows to add new atomic propositions on the computation tree before evaluating a CTL formula. This is tightly connected with strategy quantification, since a strategy can be encoded by decorating the computation tree with a move in each node. QCTL turns out to be much better-behaved than  $ATL_{sc}$ , and studying QCTL, which is interesting for itself, also gives better understanding of  $ATL_{sc}$ .

We proved that QCTL is (in some sense) as expressive as MSO; that it can be turned in prenex normal form; that it admits PSPACE (resp. k-EXPTIME) model checking, depending on the semantics, and that in some cases, its satisfiability problem is decidable [LM13a].

Based on these results, we obtained a uniform treatment of  $ATL_{sc}$  model-checking problem, via a translation to QCTL. We also proved that while  $ATL_{sc}$  satisfiability is undecidable, it is decidable when restricting to turn-based structures, or when the set of moves and agents is given as input [LM13b].

**Participants:** Nicolas Markey (CNRS)

**References:** [LM13b, LM13a]

## 4 Conclusions and perspectives

The quest for adequate solutions concepts for collective adaptive systems is a long and interesting challenge. We have already obtained several promising results: including algorithm for secure equilibria in the two-player framework,

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[DLM10] Arnaud Da Costa, François Laroussinie, and Nicolas Markey. ATL with strategy contexts: Expressiveness and model checking. In Kamal Lodaya and Meena Mahajan, editors, *Proceedings of the 30th Conference on Foundations of Software Technology and Theoretical Computer Science (FSTTCS'10)*, volume 8 of *Leibniz International Proceedings in Informatics*, pages 120–132. Leibniz-Zentrum für Informatik, December 2010.

existence results for secure equilibria in the multiplayer framework. We have also introduced new concepts of equilibria: the doomsday equilibrium and the  $k$ -fault tolerant Nash Equilibrium. Finally, we have investigated the alternative approach which consist in defining the interesting solution concepts for collective adaptive systems through temporal logics.

Although we have already made several steps on our quest, numerous questions remain open, and there is room future work. We identify below several tracks that we intend to investigate:

- We would like to push as far of possible the approach of the *suspect game* presented in [Bre12]. More precisely, we would like to define an abstract concept of equilibrium for which, one could build an equivalent of the *suspect game*, allowing to obtain decidability result for these abstract equilibria.
- Now that the existence of secure equilibria in the multiplayer framework has been proved. One can wonder how to compute “interesting” secure equilibria. Indeed the existence proof proposed in [DPFK<sup>+</sup>14] is non constructive.
- The notion of *sub-game perfect secure equilibrium* [BBDG12] has been introduced to remove the inconsistent behaviours known as *unbelievable threats* while coping with the advantages of the secure equilibria. Few results are known about this concept, for instance, there is no existence result in the multiplayer case.
- In [KLŠT12] a construction of a Büchi automaton accepting all Nash

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- [Bre12] Romain Brenguier. *Équilibres de Nash dans les Jeux Concurrents - Applications aux Jeux Temporisés*. Ph.D. Thesis, Laboratoire Spécification et Vérification, ENS Cachan, France, 2012.
- [BBDG12] Thomas Brihaye, Véronique Bruyère, Julie De Pril, and Hugo Gimbert. Subgame perfection for equilibria in quantitative reachability games. In Lars Birkedal, editor, *Proceedings of the 15th International Conference on Foundations of Software Science and Computation Structure (FoSSaCS'12)*, volume 7213 of *Lecture Notes in Computer Science*, pages 286–300. Springer-Verlag, March 2012.
- [KLŠT12] Miroslav Klimoš, Kim Gulstrand Larsen, Filip Štefaňák, and Jeppe Thaarup. Nash equilibria in concurrent priced games. In Adrian Horia Dediu and Carlos Martín-Vide, editors, *Proceedings of the 6th International Conference on Language and Automata Theory and Applications (LATA'12)*, volume 7183 of *Lecture Notes in Computer Science*, pages 363–376. Springer-Verlag, May 2012.

equilibria has been proposed. This construction offers a nice and compact representations of the equilibria of a given systems and allows to discriminate “*interesting equilibria*” such as Pareto-optimal equilibria. It would be nice to generalise this approach to other classes of equilibria.

## 5 List of publications

- [BMR14] Véronique Bruyère, Noémie Meunier, and Jean-François Raskin. Secure equilibria in weighted games. *CoRR*, abs/1402.3962, 2014.
- [CDFR14] Krishnendu Chatterjee, Laurent Doyen, Emmanuel Filiot, and Jean-François Raskin. Doomsday equilibria for omega-regular games. In *VMCAI*, volume 8318 of *Lecture Notes in Computer Science*, pages 78–97. Springer, 2014.
- [DPFK<sup>+</sup>14] Julie De Pril, János Flesch, Jeroen Kuipers, Gijs Schoenmakers, and Koos Vrieze. Existence of secure equilibrium in multi-player games with perfect information. *Submitted*, 2014.
- [LM13a] François Laroussinie and Nicolas Markey. Quantified CTL: expressiveness and complexity. Research Report LSV-13-07, Lab. Spécification & Vérification, ENS Cachan, France, April 2013.
- [LM13b] François Laroussinie and Nicolas Markey. Satisfiability of at1 with strategy contexts. In Gabriele Puppis and Tiziano Villa, editors, *Proceedings of the 4th International Symposium on Games, Automata, Logics and Formal Verification (GandALF'13)*, volume 119 of *Electronic Proceedings in Theoretical Computer Science*, pages 208–223, August 2013.