Buffered Simulation Games for Büchi Automata

M. Hutagalung   M. Lange   E. Lozes

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Language inclusion for Büchi automata

The problem

Given two non-deterministic Büchi automata (NBA) A, B, does it hold that

$$L(A) \subseteq L(B)?$$

L.I. in model-checking

- LTL model-checking
- regular model-checking
- size-change termination analysis
- ...

A hard problem

- PSPACE-complete
- already for trace inclusion (for two LTS)
- sometimes even undecidable (e.g. TA)

Recent advances on language inclusion

- antichain algorithms [Doyen *et al*]
- bisimulation up to techniques [Bonchi and Pous]
- Ramsey or antichain algorithms enhanced with simulation preorder [Holik *et al*]
- automata minimization
  - using SAT solving [Ehlers]
  - quotienting, pruning [Clemente and Mayr]
- `http://www.languageinclusion.ord`

Our goal: design better simulations $\Rightarrow$ get better algorithms

Simulations step by step

Why simulation matters

- under-approximates language inclusion
- can be used for automata minimization
- computing simulation is typically fast
  in time $O(|A| \cdot |B|)$ for two NFA A, B

Looking for better simulations

- delayed simulation [Etessami, Wilke, and Schuller]
  $\Rightarrow$ NBA quotient is language equivalent
- multi-pebble simulation [Etessami]
  $\Rightarrow$ better approximates language inclusion
- multi-letter simulation
  [Clemente and Mayr] [Hutagalung *et al*]
  $\Rightarrow$ sometimes faster than multi-pebble

Fair simulation

two players, infinite game

- Spoiler moves a pebble in A, Duplicator in B
- in every round:
  1. Spoiler chooses a letter a
  2. Spoiler moves his pebble along an a transition
  3. Duplicator moves her pebble along an a transition

- if a player gets stuck, he/she looses
- otherwise, two infinite runs
- Fairness condition: Duplicator wins if
  - either her run is accepting,
  - or Spoiler's one is not.

k-letters fair simulation

two players, infinite game

- Spoiler moves a pebble in A, Duplicator in B
- in every round:
  1. Spoiler chooses k letters $a_1, \ldots, a_k$
  2. Spoiler moves his pebble along transitions $a_1, \ldots a_k$
  3. Duplicator moves her pebble along transitions $a_1, \ldots a_k$

- if a player gets stuck, he/she looses
- otherwise, two infinite runs
- Fairness condition: Duplicator wins if
  - either her run is accepting,
  - or Spoiler's one is not.

Example

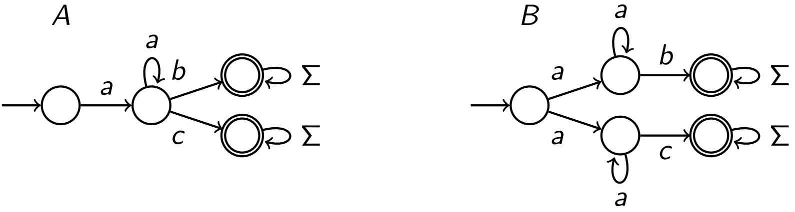
- $(A) \subseteq L(B)$ but A is not simulated by B
- A is 2-letters simulated by B

k-letters simulations in practice

- cheap to compute for small k
- on most benchmarks, significantly better than 1-letter simulation (up to 100 times faster than Rabit v1.0)

- untractable for large k
- Duplicator looses in "easy cases"

Example



- $L(A) \subseteq L(B)$
- A is not k-letters simulated by B, for any k.

Introducing continuous simulation

players now share a FIFO buffer
in every round:

1. Spoiler chooses $a \in \Sigma$, and adds it to the buffer
2. Spoiler moves his pebble along an a transition
3. Duplicator can decide to
   - either skip her turn
   - or pop b from the buffer, and move along a b transition

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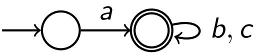
Winning condition

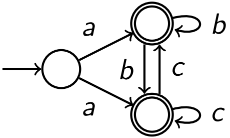
- if a player gets stuck, he/she looses
- if Spoiler makes no accepting run, he looses

otherwise,

- if Duplicator eventually only skips her turn, she looses
- otherwise, apply fairness condition on the two infinite runs

Example

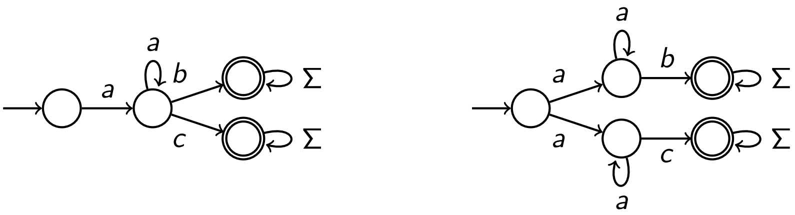




Winning strategy for Duplicator:

- skip her first turn
- at round i, play Spoiler's letter in round $i-1$

Fairness and buffer boundedness



- Spoiler must eventually leave the *a* loop
- Duplicator wins
- buffer can grow arbitrarily large

Continuous simulation and continuity

- ARuns(A): the set of accepting runs of A
- ARuns(A) is a metric space
- f : ARuns(A) $\rightarrow$ ARuns(B) is *word preserving* if $f(\rho)$ and ρ are runs over the same infinite word.

Theorem

1. $L(A) \subseteq L(B)$ iff there is a word preserving f : ARuns(A) $\rightarrow$ ARuns(B)
2. B cont. simulates A iff there is a continuous word preserving f : ARuns(A) $\rightarrow$ ARuns(B)

Consequences

- continuous simulation is a transitive relation
  - continuity is preserved by function composition
- continuous simulation is decidable in 2-EXPTIME
  - follows from [Holtmann, Kaiser and Thomas, FSTTCS 2010]
- on LTS (without fairness), the buffer can be bounded
  - for a LTS S, traces(S) is a compact space
  - on such spaces, continuous functions = Lipschitz functions
  - Lipschitz function = buffer boundedness

Exact complexity

Theorem

Continuous simulation is EXPTIME-complete.

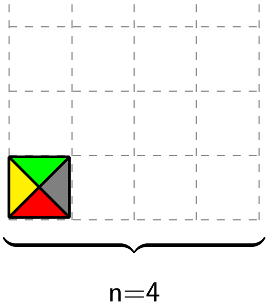
Decidability

- $w \sim_A w'$ if w, w' have the same "transition profile"
- $3^{|A|^2}$ equivalence classes
- cannot abstract buffer exact content by equiv class ...
- first introduce an equivalent game with "lassos"
- end with parity game of exponential size and index 3.

EXPTIME-hardness

Tiling game

- a set of tiles, among which one called “target”
- on every round
  - Starter chooses the first tile in the row



EXPTIME-hardness

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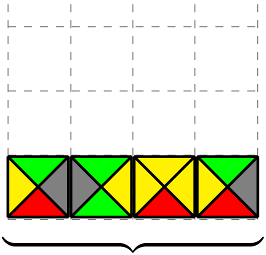
  - Completer chooses the $n-1$ other tiles of the row



n=4

EXPTIME-hardness

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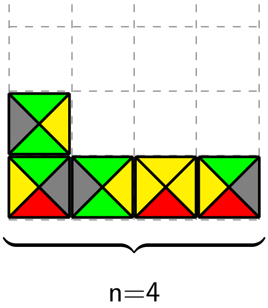

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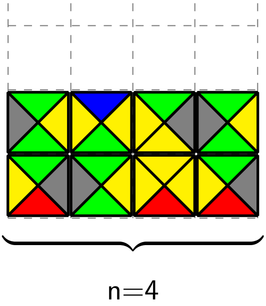

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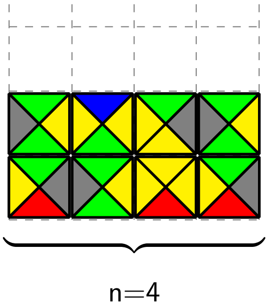


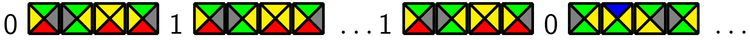
Completer wins if she eventually puts the target tile.

EXPTIME-hardness (2)

Given a tiling game G, define A_G and B_G such that

Starter wins G iff Duplicator wins contsim(A_G, B_g).

- alphabet $=$ set of tiles $+ \{0,1\}$
- Spoiler is forced to play a word of the form



- Duplicator forces Spoiler's first tile by its position
- Spoiler forces Duplicator to play by repeating a row

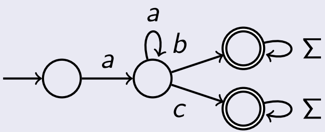
Look-ahead simulation

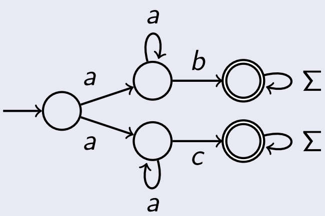
in every round:

1. Spoiler chooses $a \in \Sigma$, and adds it to the buffer
2. Spoiler moves his pebble along an a transition
3. Duplicator can decide to
   - either skip her turn
   - or flush the entire buffer, get $a_1, \ldots, a_k$, and move along $a_1, \ldots, a_k$ transitions
4. winning condition as before

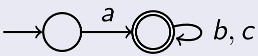
Examples

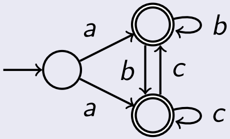
Duplicator wins





Spoiler wins





PSPACE hardness

Theorem

Look-ahead fair simulation is PSPACE hard.

Proof:

- reducing from language inclusion for NFA
- for an NFA A, define A' such that $L(A') = L(A).\#^{\omega}$ adding an extra state
- then $L(A) \subseteq L(B)$ iff B' fairly simulates A' with look-ahead

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Theorem

Look-ahead unfair simulation is PSPACE hard.

Proof: more involved, again reducing from a tiling problem.

PSPACE upper bound

Theorem

Look-ahead fair simulation is in PSPACE.

<u>Proof:</u>

- define quotient game along the same lines as before
- still get parity game G of exponential size
- however, only $O(|A| \cdot |B|)$ positions for Spoiler in G
- solve the game without generating it entirely

A word on automata minimization

Other winning condition

- fair simulation is not good for quotienting
- delayed simulation is [Etessami, Wilke, and Schuller]
- direct simulation is good for pruning

Same holds for continuous/look-ahead counterparts.

Look-ahead simulation is not transitive

- counter-example for multi-letters due to Clemente and Mayr
- carry over look-ahead simulations
- consequence : quotienting is not idempotent, can be repeated

Conclusion

Contribution

- we introduced two new simulations
- we established their decidability and exact complexity
- continuous simulation is mathematically appealing

Perspectives

- high complexities $\Rightarrow$ probably not suitable for NBA
- interesting for timed automata?