Weighted Timed Games: Positive Results with Negative Costs

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Smart Houses on a Grid (Jadevej Case)

Eight houses
Electric local grid

Each house:

- Solar panels
- Electric heating
- Storage of energy

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 Solar panel ON

- Selling energy: +2€/t.u.
- Consumption: 0€/t.u.
- Storing energy: 0€/t.u.

 Solar panel OFF

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- Selling energy: +1€/t.u.
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+ fixed cost to start selling or buying energy

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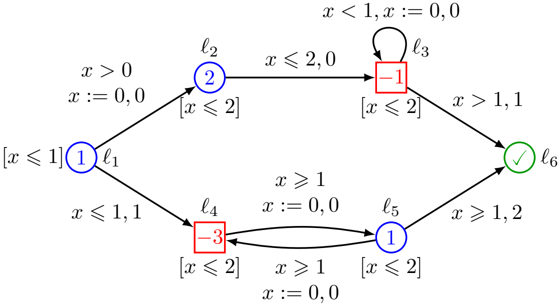
Our contribution: Synthesize optimal behaviors in each phase by solving weighted timed games with a limited number of distinct rates

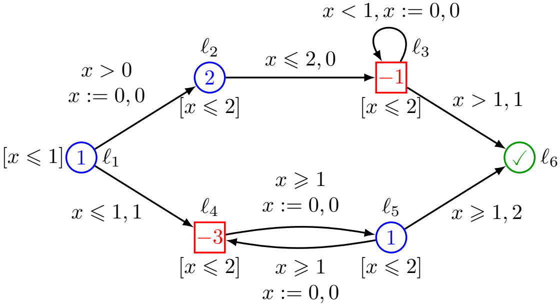
Weighted Timed Games



Timed Automaton
with partition of states
between 2 players
+ reachability objective
+ rates in locations
+ costs over transitions

Semantics in terms of
infinite game with weights

Weighted Timed Games



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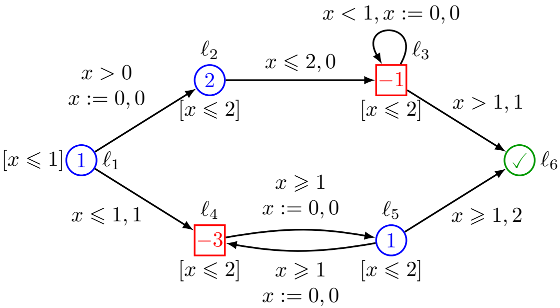

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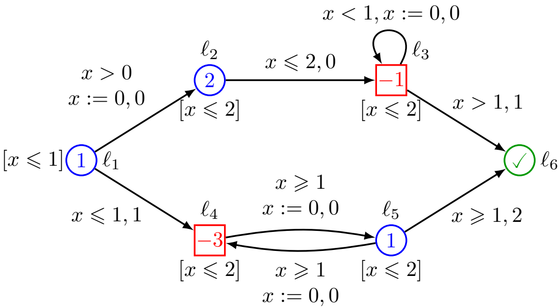
$(\ell_1, 0) \xrightarrow{0.4,\searrow} (\ell_4, 0.4)$

Weighted Timed Games



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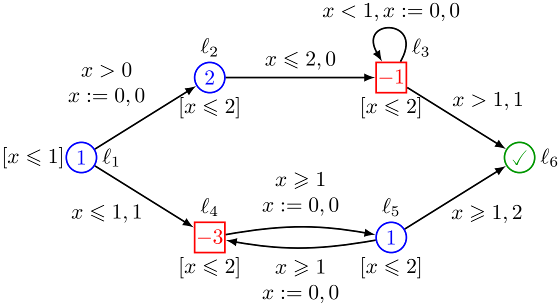
Weighted Timed Games



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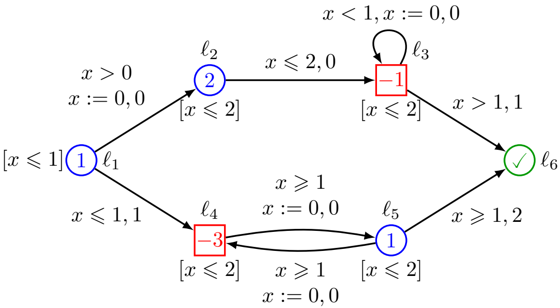

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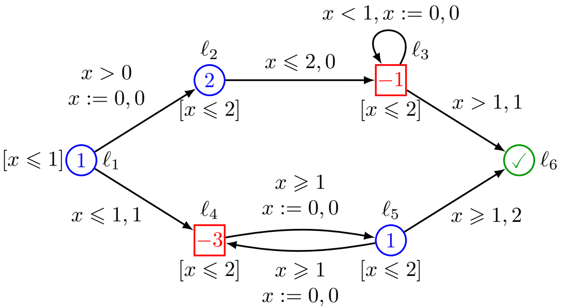
$$0.4 + 1 \quad -3 \times 0.6 \quad +1.5 \quad -3 \times 1.1 \quad +2 \times 2 + 2 \quad = 3.8$$

Weighted Timed Games



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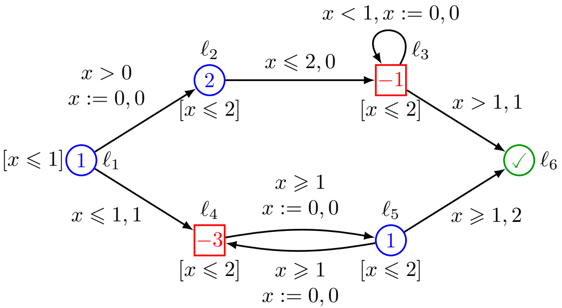
$$(\ell_1, 0) \xrightarrow[0.4+1]{0.4,\searrow} (\ell_4, 0.4) \xrightarrow[-3\times 0.6]{0.6,\rightarrow} (\ell_5, 0) \xrightarrow[+1.5]{1.5,\leftarrow} (\ell_4, 0) \xrightarrow[-3\times 1.1]{1.1,\rightarrow} (\ell_5, 0) \xrightarrow[+2\times 2+2]{2,\nearrow} (\checkmark, 2) \qquad = 3.8$$

$$(\ell_1, 0) \xrightarrow[0.2]{0.2,\nearrow} (\ell_2, 0) \xrightarrow[+0.9]{0.9,\rightarrow} (\ell_3, 0.9) \xrightarrow[-0.2]{0.2,\circlearrowleft} (\ell_3, 0) \xrightarrow[-0.9]{0.9,\circlearrowleft} (\ell_3, 0) \quad \cdots \qquad = +\infty \ (\checkmark \text{ not reached})$$

Weighted Timed Games



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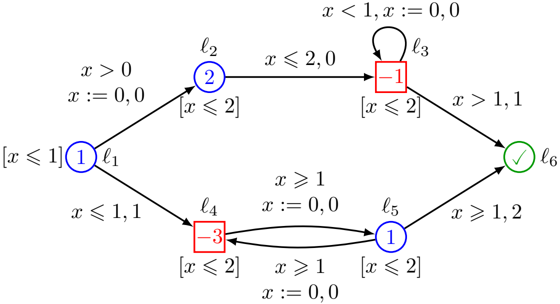
$$0.2 \quad +0.9 \quad -0.2 \quad -0.9 \quad \cdots \quad = +\infty \ (\checkmark \text{ not reached})$$

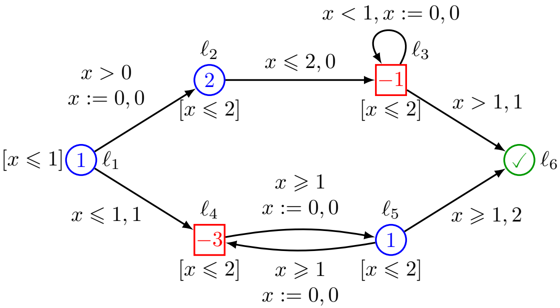
$$\text{Weight of a play: } \begin{cases} +\infty & \text{if } \checkmark \text{ not reached} \\ \text{total payoff} & \text{otherwise} \end{cases}$$

Strategies and objectives



Strategy for each player: mapping of finite runs to a delay and an action

Strategies and objectives



Strategy for each player: mapping of finite runs to a delay and an action

Goal of player ○: reach ✓ **and** minimize the cost

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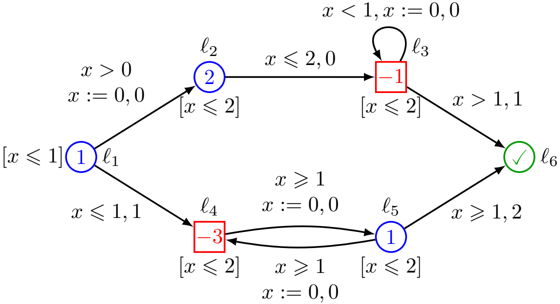

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Main object of interest:

$$\overline{\mathsf{Val}}(\ell, v) = \inf_{\sigma_\bigcirc \in \mathsf{Strat}_\bigcirc} \sup_{\sigma_\square \in \mathsf{Strat}_\square} \mathsf{Wt}(\mathsf{Play}((\ell, v), \sigma_\bigcirc, \sigma_\square)) \in \mathbf{R} \cup \{-\infty, +\infty\}$$

What player ○ can guarantee as a payoff? and design *good* strategies

State of the art

Decision problem: does there exist a strategy for player ○ ensuring a weight not greater than a given constant?

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- One-player case (**Weighted timed automata**): optimal reachability problem is PSPACE-complete
  - Algorithm based on region abstraction [Bouyer, Brihaye, Bruyère, and Raskin, 2007];
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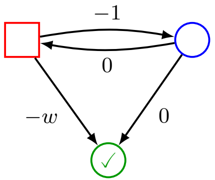
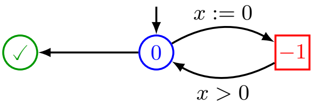
This talk: **One-clock weighted timed games with negative weights**

Why things are complex with negative weights? (even in weighted untimed finite games)

- Value $-\infty$: detection is as hard as mean-payoff. No hope for complexity better than $\mathbf{NP} \cap \mathbf{co\text{-}NP}$, or pseudo-polynomial
- Memory complexity
  - Player ○ needs memory

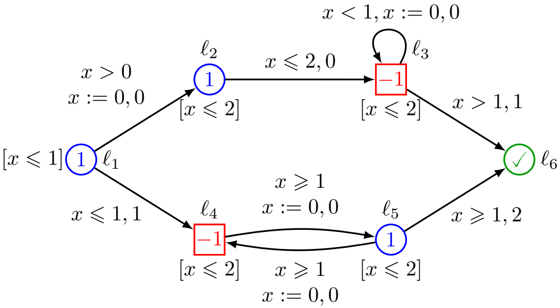


  - Player □ needs infinite memory in weighted timed games



One-clock Binary Weighted Timed Games (1BWTG)

Assumption: rates of locations $\{p^-, p^+\}$ included in $\{0, +d, -d\}$ ($d \in \mathbf{N}$) (no assumption on weights of transitions)



- Techniques of [Bouyer, Cassez, Fleury, and Larsen, 2004, Alur, Bernadsky, and Madhusudan, 2004] not applicable, e.g., because of Zeno weights cycles
- Exponential algorithms of [Bouyer, Larsen, Markey, and Rasmussen, 2006b, Rutkowski, 2011, Hansen, Ibsen-Jensen, and Miltersen, 2013] not working because of presence of negative weights

Results

Theorem:

- Computation of the value $\overline{\mathsf{Val}}(\ell, v)$ of states of a 1BWTG in pseudo-polynomial time
- Synthesis of ε-optimal strategies for player $\bigcirc$ in pseudo-polynomial time

Theorem: Non-negative case

In case of a 1BWTG with only non-negative weights, all complexities drop down to polynomial.

First idea: symetrize the point of view

Value for player $\bigcirc$: $\overline{\mathsf{Val}}(\ell, v) = \inf_{\sigma_\bigcirc \in \mathsf{Strat}_\bigcirc} \sup_{\sigma_\Box \in \mathsf{Strat}_\Box} \mathsf{Wt}(\mathsf{Play}((\ell, v), \sigma_\bigcirc, \sigma_\Box))$

Value for player $\Box$: $\underline{\mathsf{Val}}(\ell, v) = \sup_{\sigma_\Box \in \mathsf{Strat}_\Box} \inf_{\sigma_\bigcirc \in \mathsf{Strat}_\bigcirc} \mathsf{Wt}(\mathsf{Play}((\ell, v), \sigma_\bigcirc, \sigma_\Box))$

How to compare them? $\underline{\mathsf{Val}}(\ell, v) \leqslant \overline{\mathsf{Val}}(\ell, v)$

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How to compare them? $\underline{\mathsf{Val}}(\ell, v) \leqslant \overline{\mathsf{Val}}(\ell, v)$

Theorem: (continued)

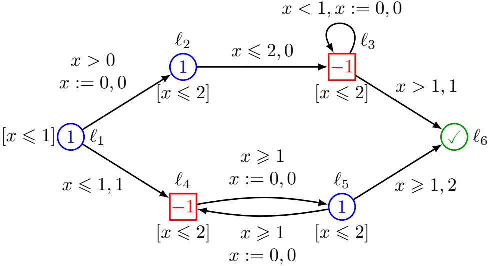
- 1BWTGs are determined: $\underline{\mathsf{Val}}(\ell, v) = \overline{\mathsf{Val}}(\ell, v)$
- Synthesis of ε-optimal strategies for player $\Box$ in pseudo-polynomial time (and polynomial in case of non-negative weights)

Sketch of proof

1. **Reduce the space of strategies in the 1BWTG**: restrict to uniform strategies w.r.t. timed behaviors
2. **Build a weighted finite games $\mathcal{G}$** based on a refinement of the region abstraction
3. **Study $\mathcal{G}$**
4. **Lift results of $\mathcal{G}$ to the original 1BWTG**

1. Reduce the space of strategies

Intuition: no need for both players to play far from boundaries of regions



Regions: $\{0\}, (0,1), \{1\}, (1,2), \{2\}, (2,+\infty)$

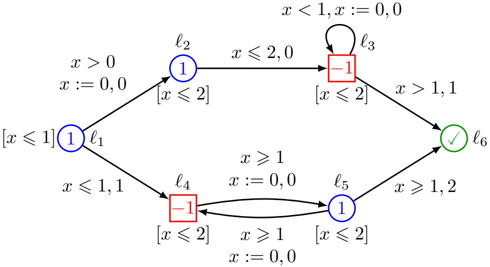
Player ○ wants to leave as soon as possible a state with rate p^+, and wants to stay as long as possible in a state with rate p^-: so, he will always play η-close to a boundary...

Lemma:

Both players can play arbitrarily close to boundaries w.l.o.g., i.e., for every η

$$\underline{\mathsf{Val}}^{\eta}(\ell, v) \leqslant \underline{\mathsf{Val}}(\ell, v) \quad \leqslant \quad \overline{\mathsf{Val}}(\ell, v) \leqslant \overline{\mathsf{Val}}^{\eta}(\ell, v)$$

2. Weighted finite game abstraction



η-regions: $\{0\}, (0, \eta), (1-\eta, 1), \{1\}, (1, 1+\eta), (2-\eta, 2), \{2\}, (2, +\infty)$

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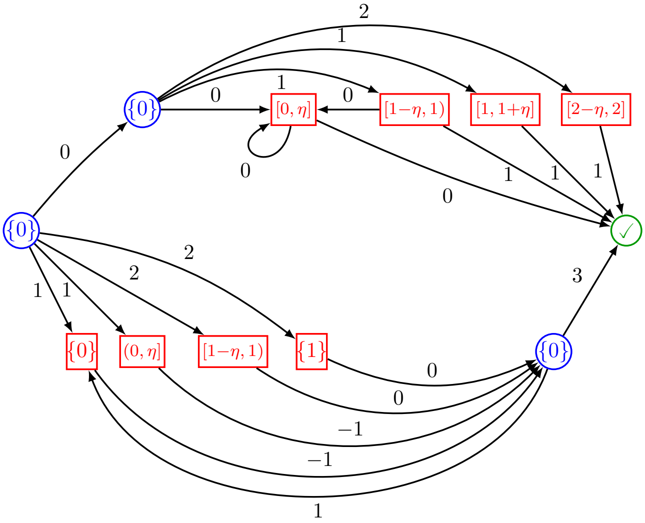
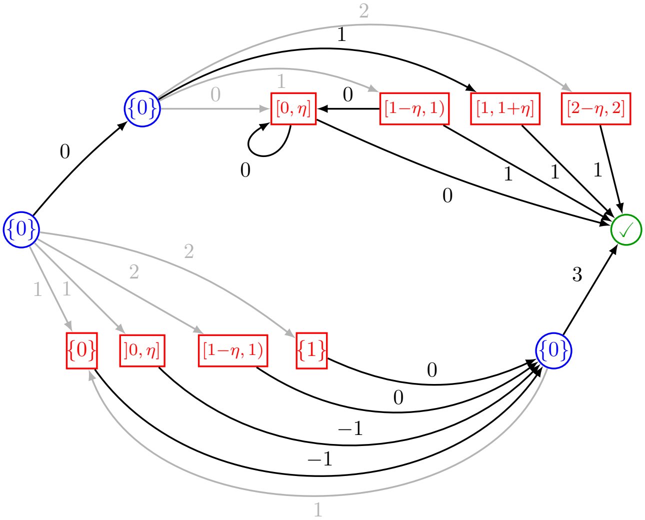


3. Study $\mathcal{G}$: values and optimal strategies



Optimal value: $\mathsf{Val}_{\mathcal{G}}(\ell_1, \{0\}) = +2$ (for both players)

4. Lift results of $\mathcal{G}$ to the original 1BWTG

Reconstruct strategies in the 1BWTG from optimal strategies of $\mathcal{G}$

Lemma:

For all $\varepsilon > 0$, there exists $\eta > 0$ such that:

$$\mathsf{Val}_{\mathcal{G}}(\ell, \{0\}) - \varepsilon \leqslant \underline{\mathsf{Val}}^{\eta}(\ell, 0) \leqslant \underline{\mathsf{Val}}(\ell, 0) \leqslant \overline{\mathsf{Val}}(\ell, 0) \leqslant \overline{\mathsf{Val}}^{\eta}(\ell, 0) \leqslant \mathsf{Val}_{\mathcal{G}}(\ell, \{0\}) + \varepsilon$$

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- So $\underline{\mathsf{Val}}(\ell, 0) = \overline{\mathsf{Val}}(\ell, 0)$, i.e., determination
- ε-optimal strategies for both players
  - Finite memory for player ○, because finite memory in weighted finite games
  - Infinite memory for player □ (even though memoryless in weighted finite games), because it needs to ensure convergence of its differences between the 1BWTG and $\mathcal{G}$
- Overall complexity: pseudo-polynomial (polynomial if non-negative weights) in the size of $\mathcal{G}$, which is polynomial in the 1BWTG (because 1 clock)

Summary and Future Work

Results

- 1BWTGs are determined: $\underline{\mathsf{Val}}(\ell, v) = \overline{\mathsf{Val}}(\ell, v)$
- Computation of the values in pseudo-polynomial time (and polynomial in case of non-negative weights)
- Synthesis of ε-optimal strategies for both players in pseudo-polynomial time (and polynomial in case of non-negative weights)
- Strategy complexity: finite memory for player $\bigcirc$, infinite memory for player $\square$

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Other results obtained in this context: undecidability results due to the presence of negative weights...

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- Implementation and test of this algorithm for real instances
- Extensions to a richer model of priced timed games with negative weights: careful since players may need to play far from boundaries in case of 2 clocks, or 1 clock and 3 distinct rates...
- Consider other objectives, e.g., timed bounded restrictions, leading to decidability in some cases

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