Infinite-state energy games

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Energy games ∼ simulation

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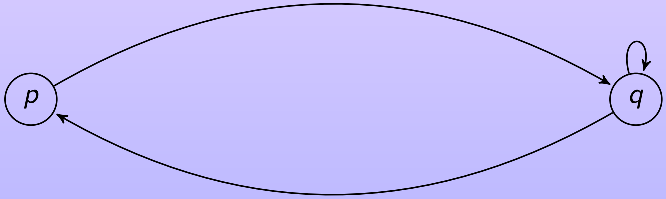
Plan

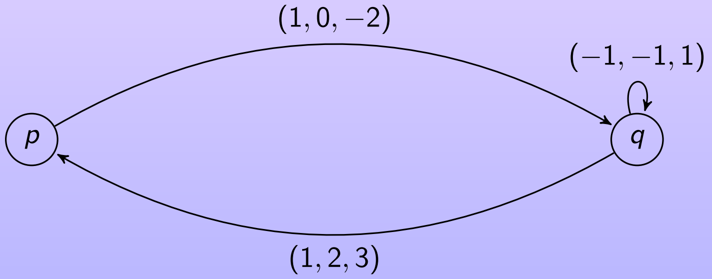
1. Basic definitions
2. Results
3. Simulation
4. Simulation $\sim$ Energy games
5. Conclusions

- Control graph.

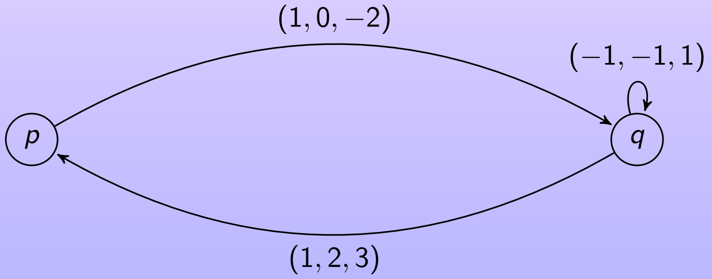


VASS with states

- Control graph.
- Edges labelled with vectors of length k.

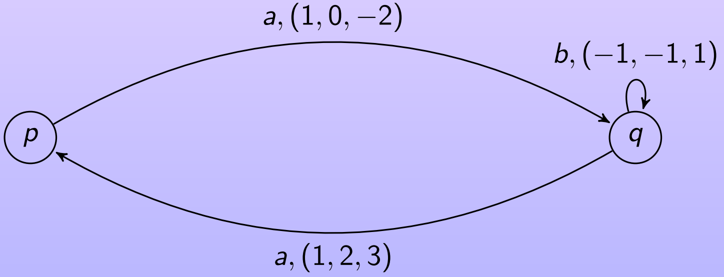


- Control graph.
- Edges labelled with vectors of length k.
- Configurations are pairs $(\textit{state}, \ \textit{vector} \in \mathbb{N}^k)$.

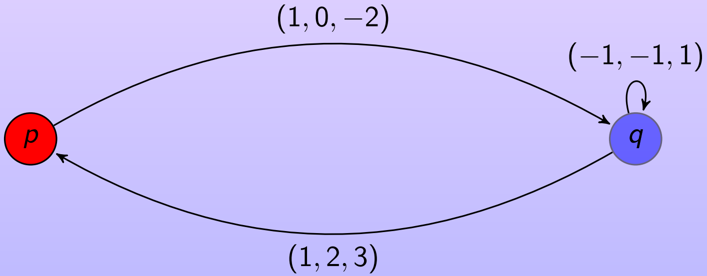


VASS with states

- Control graph.
- Edges labelled with vectors of length k.
- Configurations are pairs $(state,\ vector \in \mathbb{N}^k)$.
- We can add labels like in automaton.



- Two players Red and Blue.
- They play on VASS with states.



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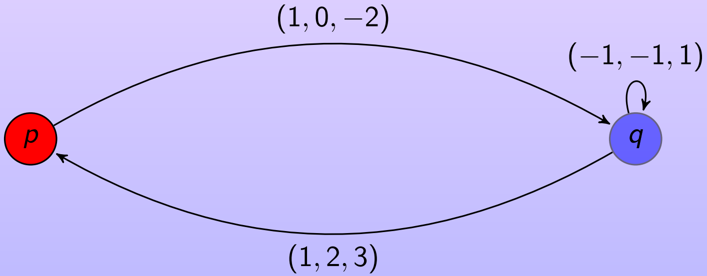

- Two players Red and Blue.
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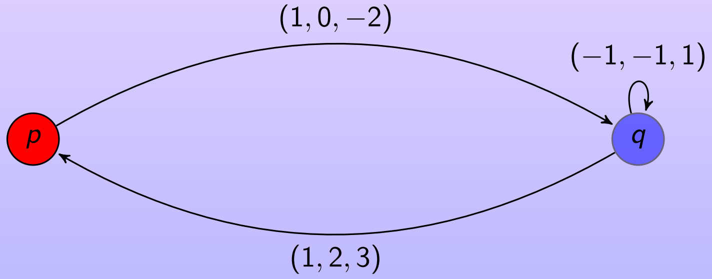
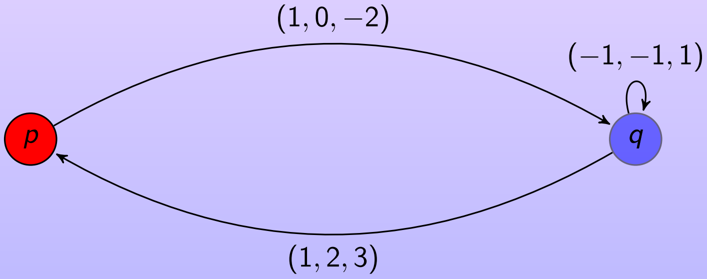


- Two players Red and Blue.
- They play on VASS with states.
- States are partitioned into red and blue.
- Configurations are pairs $(state, vector \in \mathbb{Z}^k)$.
- Energy and objectives.



There are two natural questions for energy games.

Fixed initial credit problem

Which player wins if game starts from the fixed configuration?

Unknown initial credit problem

If there exists a vector $\vec{v}$ such that the Blue player wins, in the game from the configuration $(q, \vec{v})$?

- Two players Red and Blue.
- They play on VASS with states, partitioned into red and blue.
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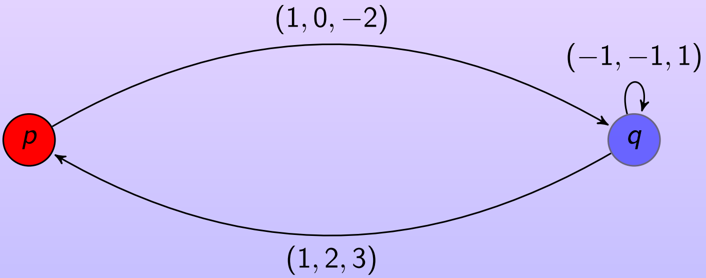

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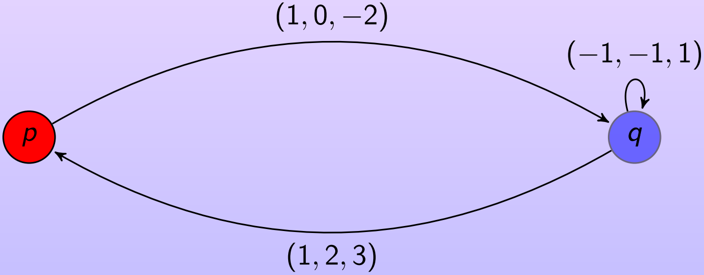
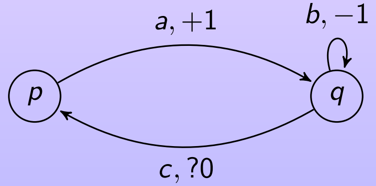


How to define Infinite-state energy games?

- States Q.
- Labelled transitions.
- Transitions with counter operations $+1, -1, ?0$.



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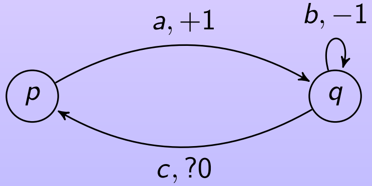
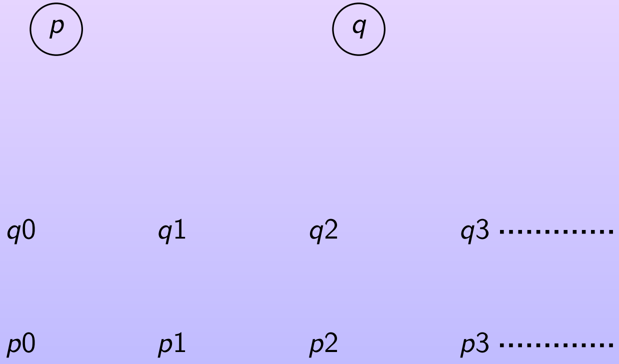


Control graph induced by one counter automaton



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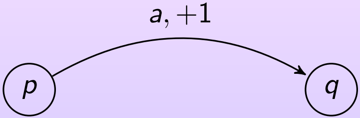

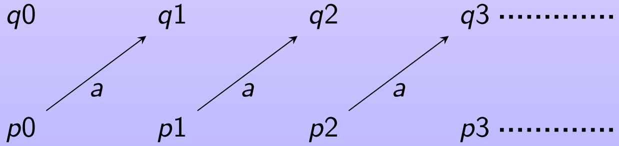

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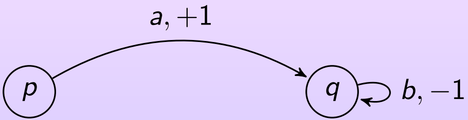

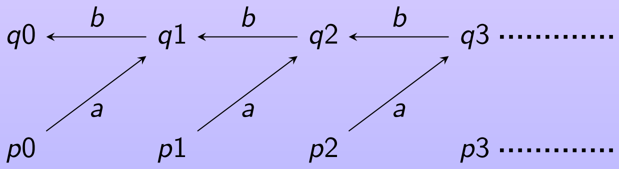

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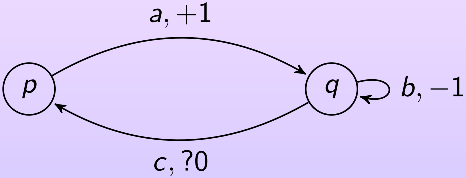
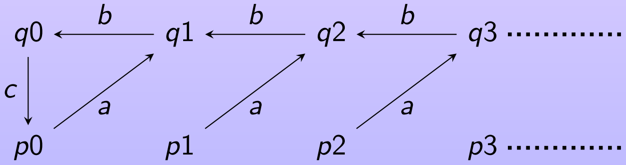
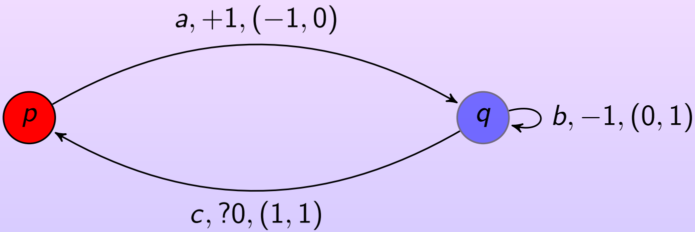
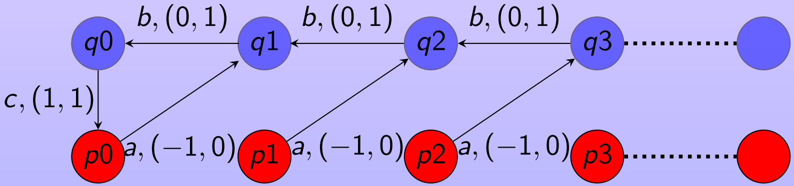
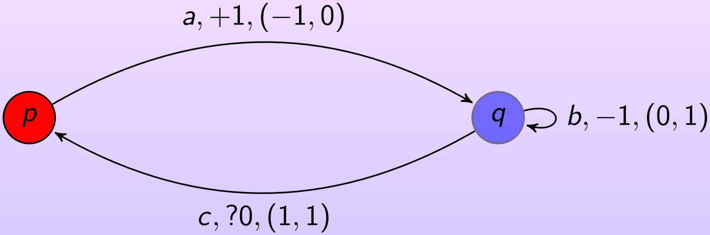


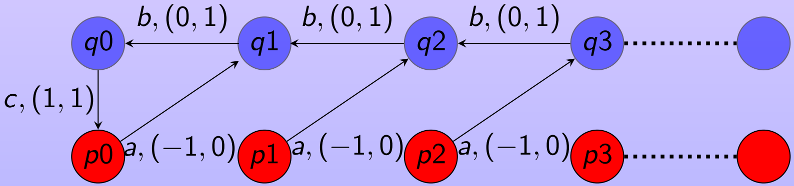


One counter automaton energy game

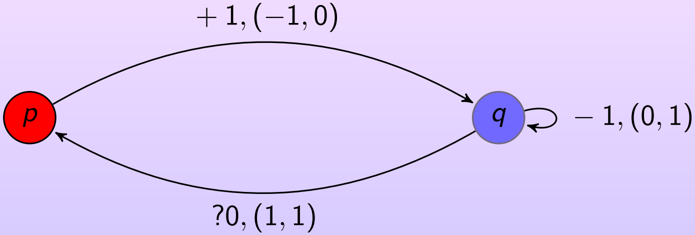


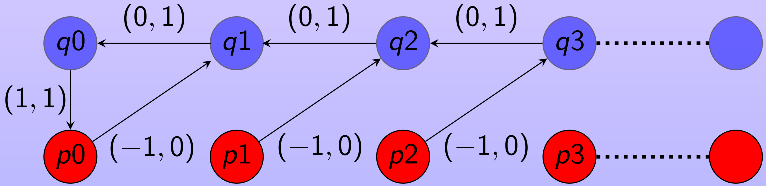






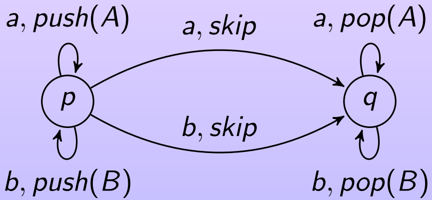
Labels are irrelevant for energy games.





Labels are irrelevant for energy games.

- States Q.
- Stack alphabet Γ.
- Transitions with stack operations *push*, *pop*, *skip*.



- Configurations are pairs $(State,\ Stack\ content)$.

Fixed and unknown initial credit problem

	Many dimensions of energy	1 energy
OCA	fixed initial credit - undecidable unknown initial credit - ???	fixed initial credit - decidable unknown initial credit - ???
PDA	undecidable	undecidable

Simulation $\sim$ Energy games

Labelled transition system

Definition (LTS)

- A graph.
- Set of vertices V may be infinite.
- Edges E labelled by elements of a finite alphabet A.

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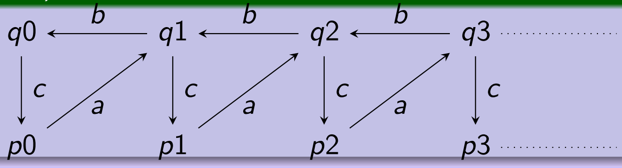
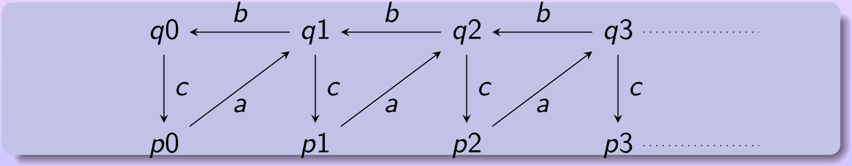
Example (LTS)



Simulation Games

. . . are played in rounds between Spoiler and Duplicator.



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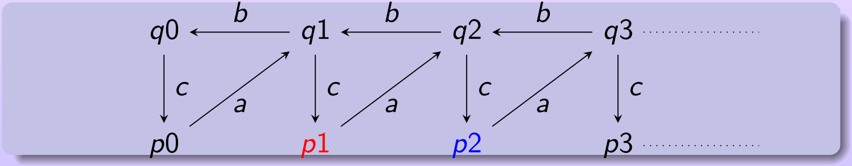
. . . are played in rounds between Spoiler and Duplicator.
Each player controls one pebble.



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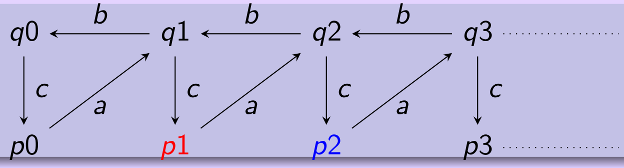

Intuitively

- Duplicator wins if he can mimic Spoiler, otherwise Spoiler is the winner.

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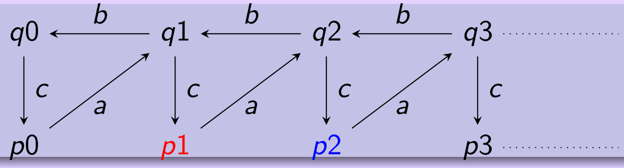

Intuitively

- Duplicator wins if he can mimic Spoiler, otherwise Spoiler is the winner.
- $p1$ is simulated by $p2$ iff Duplicator has a winning strategy in the simulation game.

Simulation Games

. . . are played in rounds between Spoiler and Duplicator.

In each round

α *vs.* β

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In each round

α *vs.* β

$\alpha \xrightarrow{a} \alpha'$

1. Spoiler moves from α.

Simulation Games

. . . are played in rounds between Spoiler and Duplicator.

In each round

$\alpha \xrightarrow{a} \alpha'$ vs. $\beta \xrightarrow{a} \beta'$

1. Spoiler moves from α.
2. Duplicator responds from β.

Simulation Games

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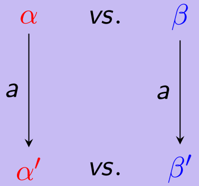
In each round



1. Spoiler moves from α.
2. Duplicator responds from β.
3. game continues from α' vs. β' .

Simulation Games

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In each round

α vs. β

$\alpha \xrightarrow{a} \alpha'$ $\qquad$ $\beta \xrightarrow{a} \beta'$

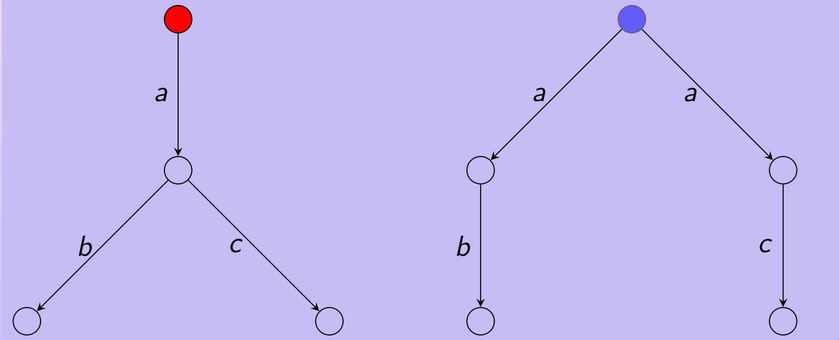
α' vs. β'

1. Spoiler moves from α.
2. Duplicator responds from β.
3. game continues from α' vs. β' .

Winning conditions:

1. If a player cannot move, the other player wins.
2. Infinite games are won by Duplicator.

Example



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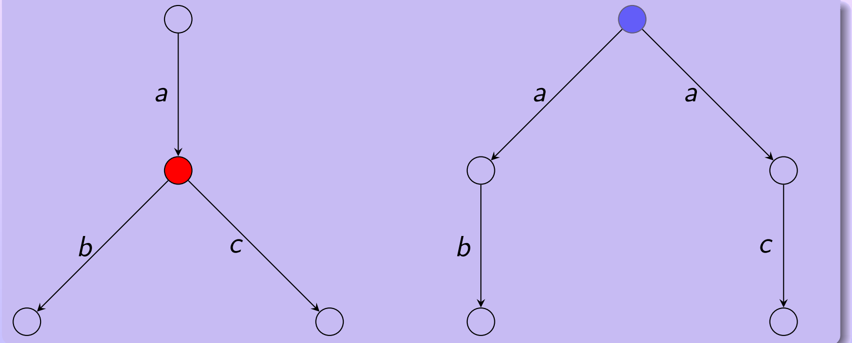

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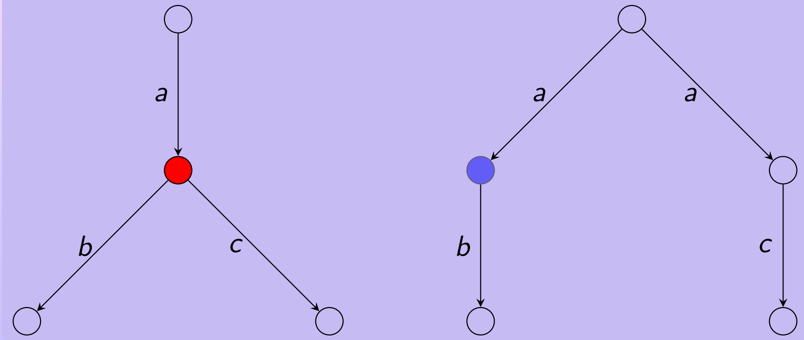

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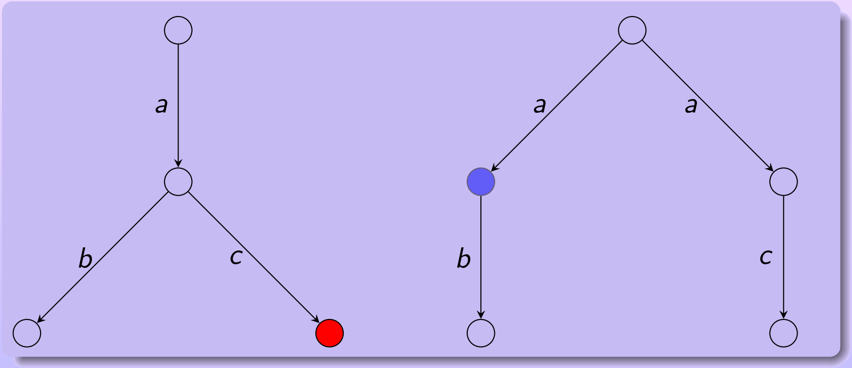

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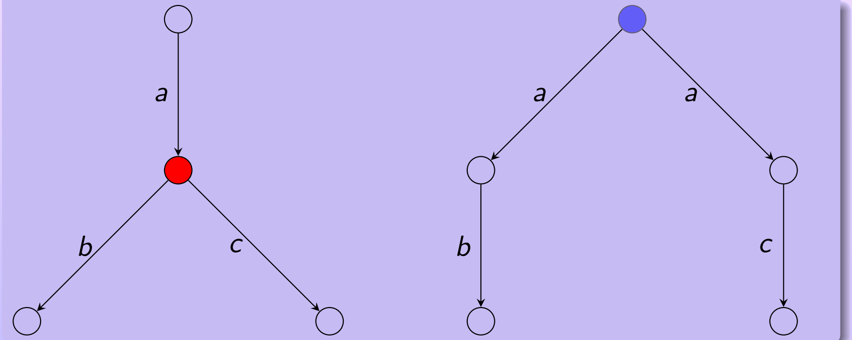

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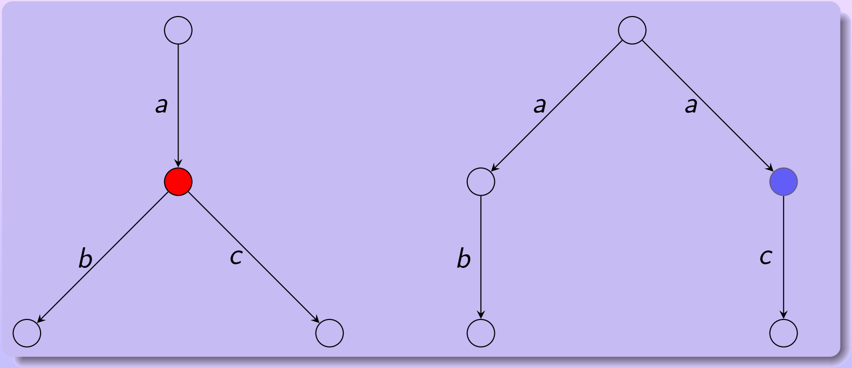

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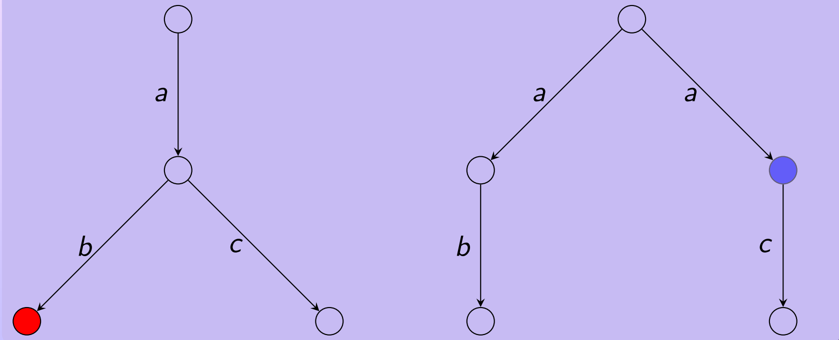

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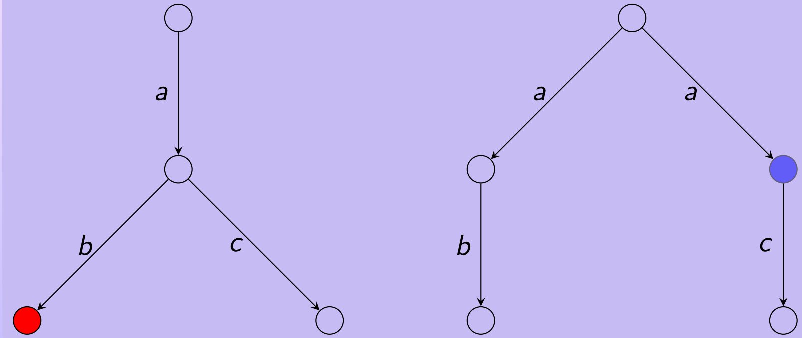

Duplicator cannot respond (Spoiler wins), so there is **no simulation.**

Theorem 1

For any n-dimensional pushdown energy game G, one can construct in logspace:

- a pushdown automaton A
- a n-dimensional VASS V,
  such that for every configuration $(q, \gamma, \vec{v})$ of a game G, Blue player wins the energy game from configuration $(q, \gamma, \vec{v})$ iff $(q, \gamma) \preceq (q, \vec{v})$.

Moreover, in the special case of a one-counter automaton energy game, the constructed automton A is a OCA.

Theorem 2

For a pushdown automaton A and an n-dimensional VASS V, one can in logspace construct an n-dimensional pushdown energy game G, such that:
for every configuration (q_0, γ) of the pushdown automaton A and every configuration $(q_1, \vec{v})$ of the VASS V,
we have $(q_0, \gamma) \preceq (q_1, \vec{v})$
iff Blue player wins the energy game from configuration $((q_0, q_1), \gamma, \vec{v})$.

Moreover, if A is a OCA then the constructed game G is a one-counter automaton energy game.

Faithful simulation

Faithful simulation

For a move in the energy game G

$$(q, \gamma, \vec{v}) \longrightarrow (q', \gamma', \vec{u})$$

in simulation game there will be a sequence of moves

$$(q, \gamma), (q, \vec{v}) \longrightarrow \cdots \longrightarrow (q', \gamma'), (q', \vec{u}).$$

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$$(q, \gamma), (q, \vec{v}) \longrightarrow \cdots \longrightarrow (q', \gamma'), (q', \vec{u}).$$

Idea: We use labels to synchronize.

Let energy game $G = (Q_0, Q_1, \Gamma, \delta, n)$.

Pushdown automaton $A = (Q_0 \cup Q_1 \cup Q_A, \Gamma, A, \delta_A)$

VASS $V = (Q_0 \cup Q_1 \cup Q_V, A, \delta_V)$

Let energy game $G = (Q_0, Q_1, \Gamma, \delta, n)$.

Pushdown automaton $A = (Q_0 \cup Q_1 \cup Q_A, \Gamma, A, \delta_A)$

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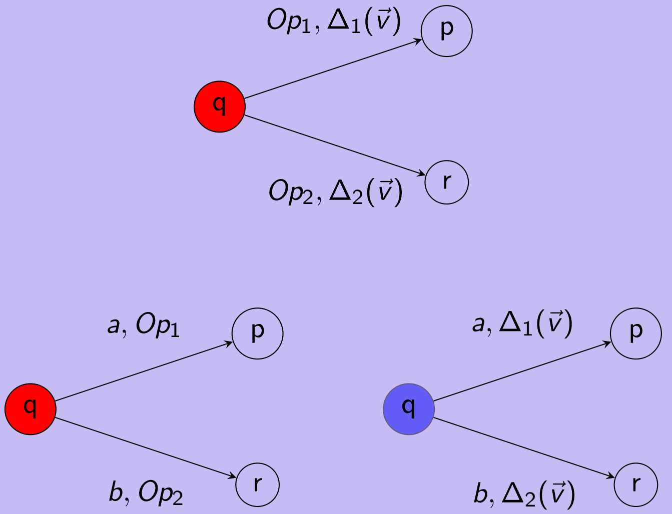
As the first step, add a unique label to every transition in δ.

States from Q_0 (Red player)



States from Q_1 (Blue player)

Question: How it is possible that Duplicator chooses a move?

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Answer: Duplikator's forcing technique.

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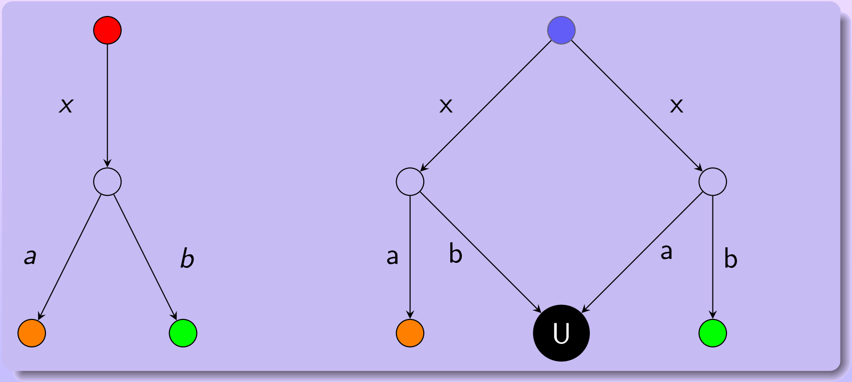
Idea:

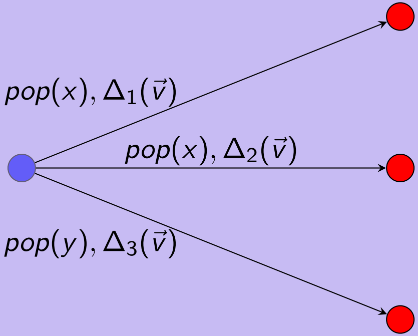
1. Spoiler makes a dummy move.
2. Duplicator makes a response in which he makes a choice.
3. Spoiler makes a move according to the Duplicator choice or she tries to cheat.
4. Duplicator **punishes** Spoiler if she cheated, or he makes a non-punishing move otherwise.

Punishing via reaching a winning state for Duplicator.

Duplicator forcing technique (picture)

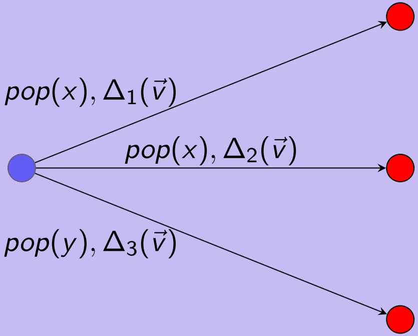


States from Q_1 (Blue player)



We have to take care about a stack symbol.

States from Q_1 (Blue player)



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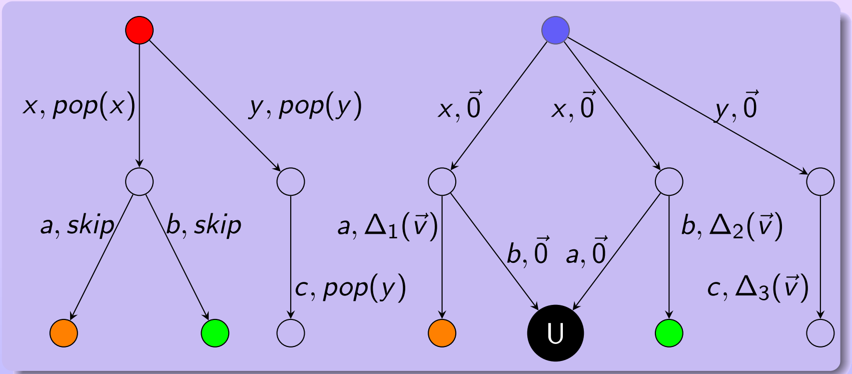

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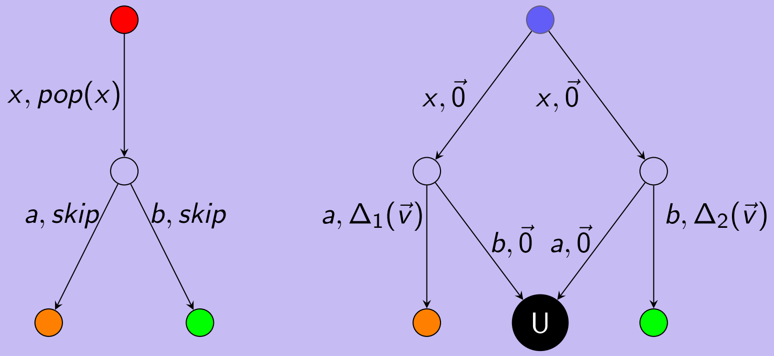

Every round of the simulation game is emulated by two steps in the energy game, one step by Red player followed by one step by Blue player.

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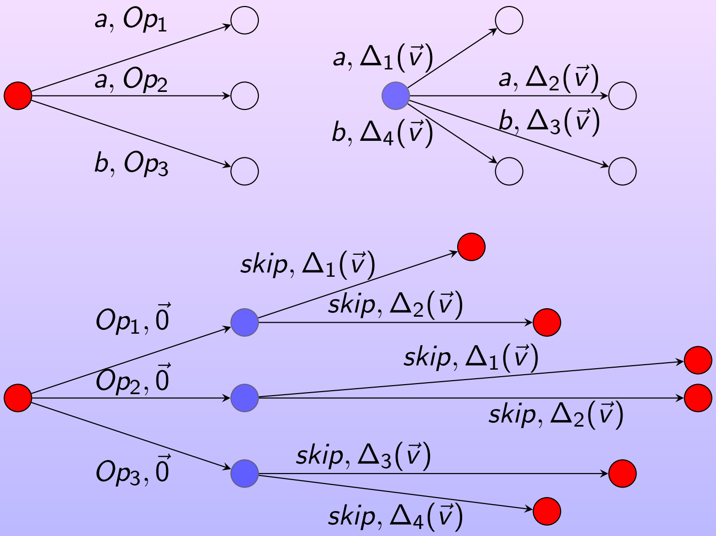
In each state of energy game we store:

- a pair of states,
- a label used by Spoiler.

Reduction $\Longleftarrow$ basic idea



Simulation

	VASS with states	one counter net
OCA	Undecidable	simulation - Decidable computing the simulation relation ???
PDA	Undecidable	Undecidable

May we extend the above proofs for other kinds of automata?

Mikołaj Bojańczyk

May we extend the above proofs for other kinds of automata?

Mikołaj Bojańczyk

- Finite automata.
- One counter nets.
- Multi-pushdown automata, with restrictions on stacks.
- Timed automata.
- Other....

Theorem (HLMT2013)

Simulation for one counter nets (OCA without zero tests) is PSPACE-complete.

One counter net is a one dimensional VASS with states.

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Theorem

Fixed initial credit problem for one counter net energy games with one dimension of energy is PSPACE-complete.

Theorem (HLMT2013)

Simulation for one counter nets (OCA without zero tests) is PSPACE-complete.

One counter net is a one dimensional VASS with states.

Theorem

Fixed initial credit problem for one counter net energy games with one dimension of energy is PSPACE-complete.

Unknown initial credit problem is in PSPACE.

Thank you.