Games with a Weak Adversary

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joint work with
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Perfect-Observation Games

Two-player games:

Existential player 1

Adversarial (universal) player 2

Graph game: $Q = Q_1 \cup Q_2$

$$\delta : (Q_1 \times A_1) \cup (Q_2 \times A_2) \to Q$$

Strategies: given finite history in Q^+, choose action in A_i

$$\sigma_1 : Q^* \cdot Q_1 \to A_1$$

$$\sigma_2 : Q^* \cdot Q_2 \to A_2$$

Perfect-Observation Games

Two-player games:

Existential player 1

Adversarial (universal) player 2

Graph game: $Q = Q_1 \cup Q_2$

$$\delta : (Q_1 \times A_1) \cup (Q_2 \times A_2) \to Q$$

Strategies: given finite history in Q^+, choose action in A_i

$$\left.\begin{array}{l} \sigma_1 : Q^* \cdot Q_1 \to A_1 \\ \sigma_2 : Q^* \cdot Q_2 \to A_2 \end{array}\right\} \Rightarrow \text{outcome}(\sigma_1, \sigma_2)$$

Perfect-Observation Games

Perfect-observation:

Both players observe the full history of the game

Parity objective Ω

Canonical specification of ω-regular objectives

Decision problem: $\exists \sigma_1 \cdot \forall \sigma_2 : \mathsf{outcome}(\sigma_1, \sigma_2) \in \Omega$

Perfect-Observation Games

Multi-player games:

Existential players 1,3,5,…

Adversarial (universal) players 2,4,6,…

Decision problem: $\exists_1 \cdot \forall_2 \cdot \exists_3 \cdot \forall_4 \cdot \exists_5 \cdots$

Perfect-Observation Games

Multi-player games:

Existential players 1,3,5,…

Adversarial (universal) players 2,4,6,…

Decision problem: $\exists_1 \cdot \forall_2 \cdot \exists_3 \cdot \forall_4 \cdot \exists_5 \cdots$

by determinacy $\exists \cdot \exists \cdots \exists \cdot \forall \cdot \forall \cdots \forall$

Perfect-Observation Games

Multi-player games:

Existential players 1,3,5,...

Adversarial (universal) players 2,4,6,...

Decision problem: $\exists_1 \cdot \forall_2 \cdot \exists_3 \cdot \forall_4 \cdot \exists_5 \cdots$

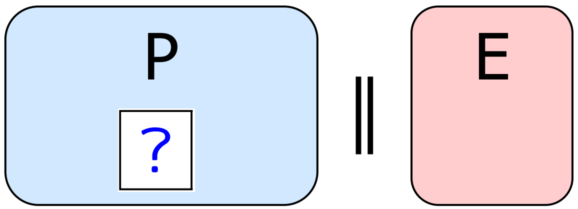
by determinacy $\underbrace{\exists \cdot \exists \cdots \exists}_{\exists_1} \cdot \underbrace{\forall \cdot \forall \cdots \forall}_{\forall_2}$

► reduces to two-player games

Partial-Observation Games

Partial-observation:

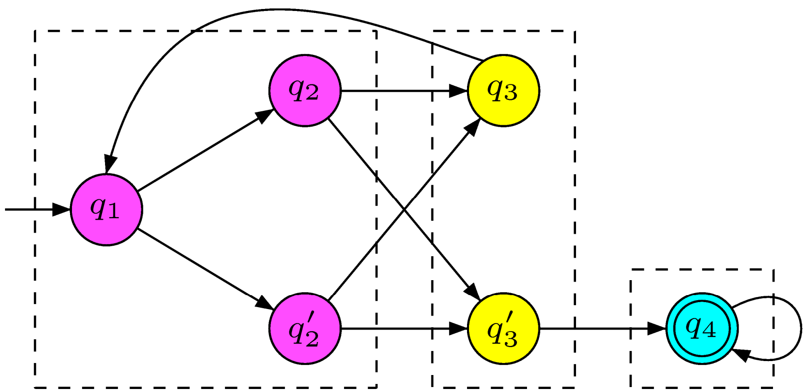
- players have partial observation of the history
- natural in synthesis, program refinement



Partial-Observation Games

Partial-observation:

Players have partial observation of the history

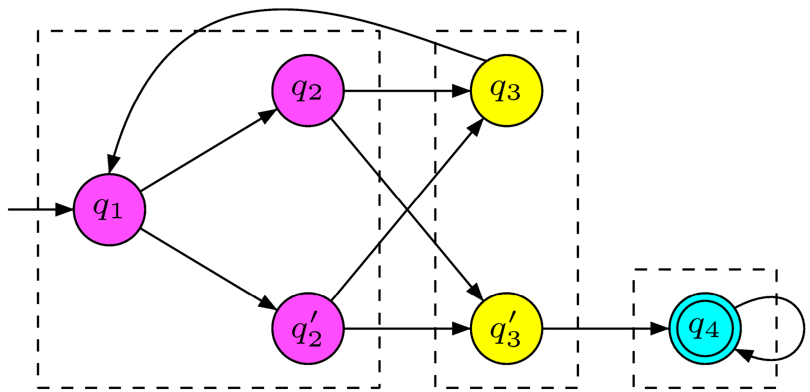


observation = color

Partial-Observation Games

Partial-observation:

Players have partial observation of the history



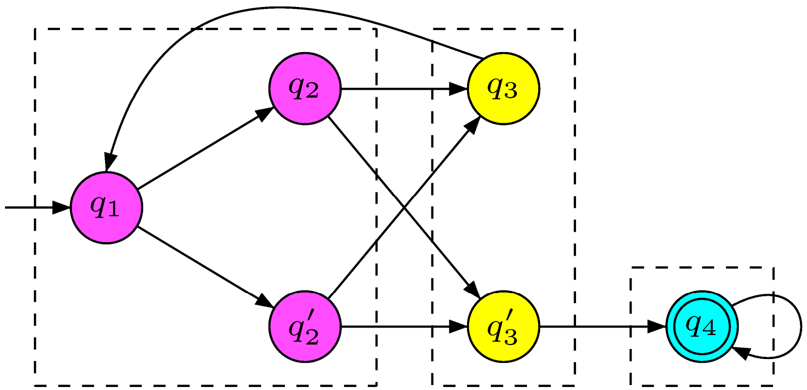
observation = color

$\mathcal{O}_i = \{\blacksquare, \blacksquare, \blacksquare\}$ $\sigma_i : \mathcal{O}_i^+ \to A_i$

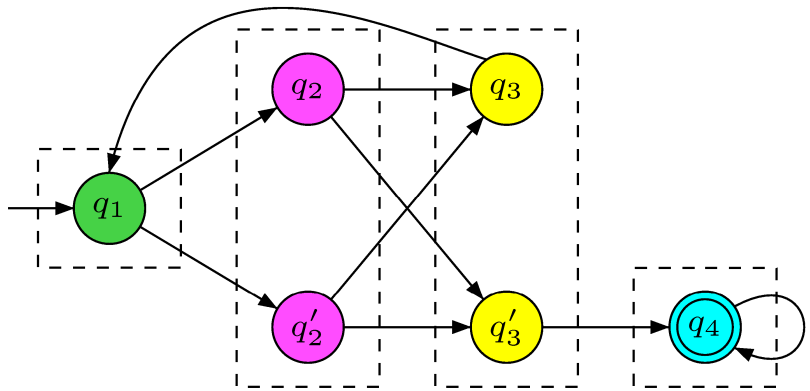
Partial-Observation Games

Partial-observation:

Players have partial observation of the history



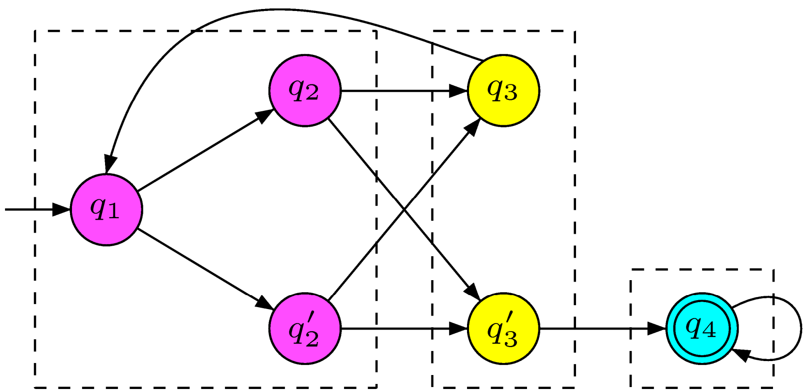
observation = color



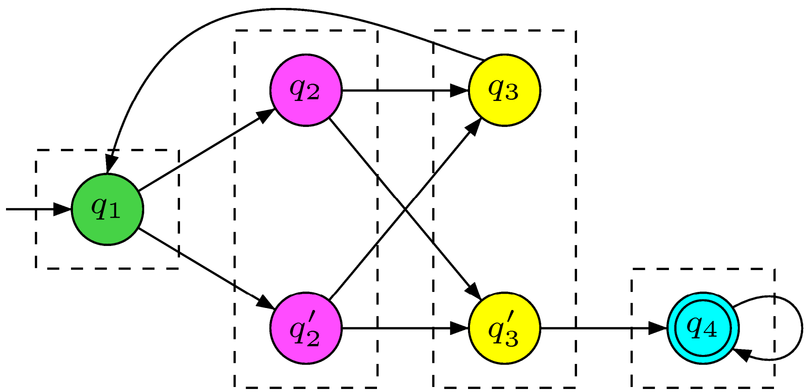
Partial-Observation Games

Partial-observation:

Players have partial observation of the history



observation = color



more informed

Partial-Observation Games

Partial-observation:

Players have partial observation of the history

Remarks:

1) $\exists \cdot \forall \neq \forall \cdot \exists$

2) $\exists \cdot \exists \cdots \exists \neq \exists$

Partial-Observation Games

Partial-observation:

Players have partial observation of the history

Remarks:

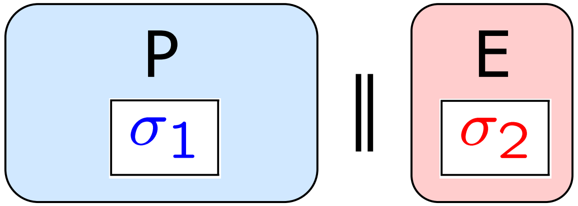
1) $\exists \cdot \forall \not\equiv \forall \cdot \exists$

2) $\exists \cdot \exists \cdots \exists \not\equiv \exists$

3) last player: partial $\equiv$ perfect

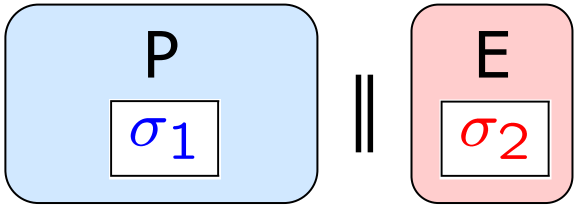
Partial-Observation Games

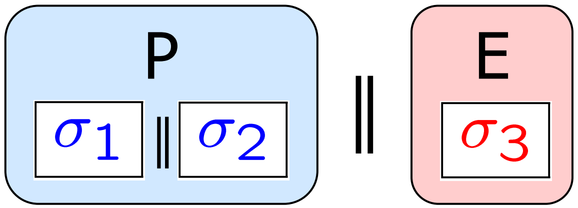
Two-player (synthesis): $\exists \sigma_1 \cdot \forall \sigma_2$



Partial-Observation Games

Two-player (synthesis): $\exists \sigma_1 \cdot \forall \sigma_2$





Three-player (distributed syn.): $\exists \sigma_1 \cdot \exists \sigma_2 \cdot \forall \sigma_3$

Partial-Observation Games

Two-player (synthesis): $\exists \sigma_1 \cdot \forall \sigma_2$

- Powerset construction (indistinguishable paths)
- stores the “belief” of player 1
- reduction to exponential-size perfect-obs. game

EXPTIME-c [Reif, CDHR06]

Three-player (distributed syn.): $\exists \sigma_1 \cdot \exists \sigma_2 \cdot \forall \sigma_3$

Partial-Observation Games

Two-player (synthesis): $\exists\sigma_1 \cdot \forall\sigma_2$

- Powerset construction (indistinguishable paths)
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EXPTIME-c [Reif, CDHR06]

Three-player (distributed syn.): $\exists\sigma_1 \cdot \exists\sigma_2 \cdot \forall\sigma_3$

Undecidable [PR89]

(except if information of the existential players forms a chain)

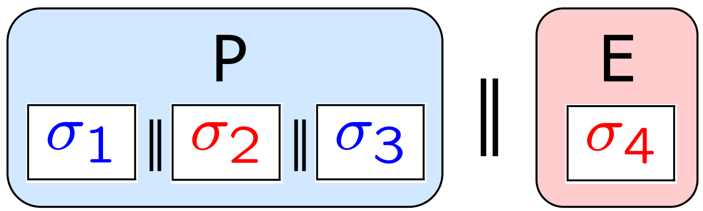
Partial-Observation Games

Two-player (synthesis):	$\exists \sigma_1 \cdot \forall \sigma_2$	EXPTIME-c
Three-player (distributed syn.):	$\exists \sigma_1 \cdot \exists \sigma_2 \cdot \forall \sigma_3$	Undecidable

Partial-Observation Games

Two-player (synthesis):	$\exists\sigma_1 \cdot \forall\sigma_2$	EXPTIME-c
Three-player (distributed syn.):	$\exists\sigma_1 \cdot \exists\sigma_2 \cdot \forall\sigma_3$	Undecidable
Muti-player (sequential syn.):	$\exists\sigma_1 \cdot \forall\sigma_2 \cdot \exists\sigma_3 \cdot \forall\sigma_4$	
	$\exists\sigma_1 \cdot \forall\sigma_2 \cdot \exists\sigma_3$	?

weak adversary



Weak adversary

Three-player (sequential syn.): $\exists\sigma_1 \cdot \forall\sigma_2 \cdot \exists\sigma_3$

- last player is perfect (wlog)
- Two difficulties:

  "Powerset" construction under scope of $\exists\sigma_1$

  Alternation of quantifiers

Weak adversary

Three-player (sequential syn.): $\exists \sigma_1 \cdot \forall \sigma_2 \cdot \exists \sigma_3$

- last player is perfect (wlog)
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  "Powerset" construction under scope of $\exists \sigma_1$

  Alternation of quantifiers

► Reduction to exponential partial-observation game

Weak adversary

Three-player (sequential syn.): $\exists\sigma_1 \cdot \forall\sigma_2 \cdot \exists\sigma_3$

$\downarrow$

$\exists\sigma_1 \cdot \underbrace{\forall\sigma_2' \cdot \exists\sigma_3'}_{\text{perfect-obs}}$

► Reduction to exponential partial-observation game

Weak adversary

Three-player (sequential syn.): $\exists\sigma_1 \cdot \forall\sigma_2 \cdot \exists\sigma_3$

$$\downarrow$$

$$\exists\sigma_1 \cdot \underbrace{\forall\sigma_2' \cdot \exists\sigma_3'}_{\text{perfect-obs}}$$

$$\downarrow$$

$$\exists\sigma_1 \cdot \exists\sigma_3' \cdot \forall\sigma_2'$$

► Reduction to exponential partial-observation game

Weak adversary

Three-player (sequential syn.): $\exists\sigma_1 \cdot \forall\sigma_2 \cdot \exists\sigma_3$

$$\downarrow$$

$$\exists\sigma_1 \cdot \underbrace{\forall\sigma_2' \cdot \exists\sigma_3'}_{\text{perfect-obs}}$$

$$\downarrow$$

$$\underbrace{\exists\sigma_1 \cdot \exists\sigma_3'}_{\exists\sigma_{13}} \cdot \forall\sigma_2'$$

► Reduction to exponential partial-observation game

Weak adversary

Three-player (sequential syn.): $\exists \sigma_1 \cdot \forall \sigma_2 \cdot \exists \sigma_3$

$$\exists \sigma_1 \cdot \underbrace{\forall \sigma_2' \cdot \exists \sigma_3'}_{\text{perfect-obs}}$$

ⓘ correct if player 1 is **less informed** than player 2

$$\underbrace{\exists \sigma_1 \cdot \exists \sigma_3'}_{\exists \sigma_{13}} \cdot \forall \sigma_2'$$

► Reduction to exponential partial-observation game

Weak adversary

Three-player (sequential syn.): $\exists \sigma_1 \cdot \forall \sigma_2 \cdot \exists \sigma_3$

exp.

$\exists \sigma_1 \cdot \underbrace{\forall \sigma_2' \cdot \exists \sigma_3'}_{\text{perfect-obs}}$

ⓘ correct if player 1 is **less informed** than player 2

$\underbrace{\exists \sigma_1 \cdot \exists \sigma_3'}_{\exists \sigma_{13}} \cdot \forall \sigma_2'$

exp.

► Reduction to exponential partial-observation game

2EXPTIME

2EXPTIME-hardness

Three-player (sequential syn.): $\exists\sigma_1 \cdot \forall\sigma_2 \cdot \exists\sigma_3$

2EXPTIME-hard ?

2EXPTIME-hardness

Three-player (sequential syn.): $\exists\sigma_1 \cdot \forall\sigma_2 \cdot \exists\sigma_3$

2EXPTIME-hard ? simulation of exp.-space alternating TM
(even for rechability)

Given <M,w> construct game: w $\in$ L(M) iff $\exists\sigma_1 \cdots$

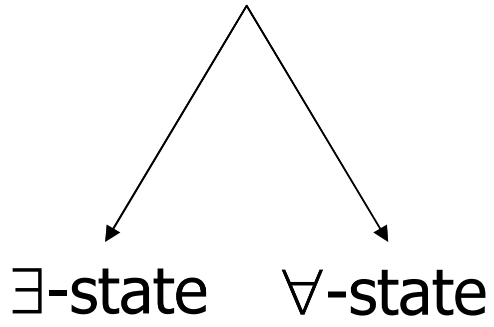
2EXPTIME-hardness

Three-player (sequential syn.): $\exists\sigma_1 \cdot \forall\sigma_2 \cdot \exists\sigma_3$

2EXPTIME-hard ? simulation of exp.-space alternating TM
(even for rechability)

Given <M,w> construct game: w $\in$ L(M) iff $\exists\sigma_1 \cdots$

of size O(|M|+|w|)



2EXPTIME-hardness

Three-player (sequential syn.): $\exists \sigma_1 \cdot \forall \sigma_2 \cdot \exists \sigma_3$

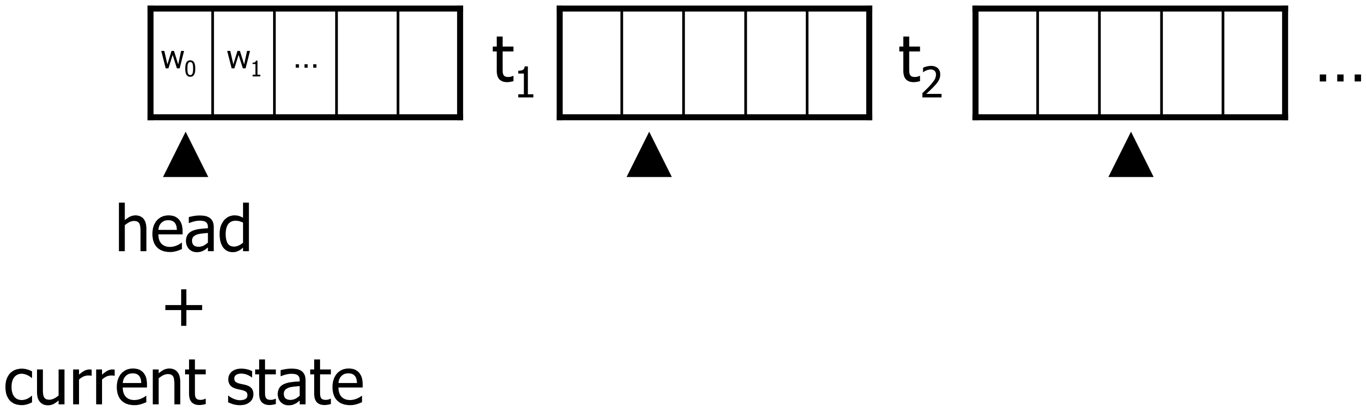
Game graph: • initialization

2EXPTIME-hardness

Three-player (sequential syn.): $\exists\sigma_1 \cdot \forall\sigma_2 \cdot \exists\sigma_3$

Game graph:

- initialization
- player 1 spells out a run of M on w:



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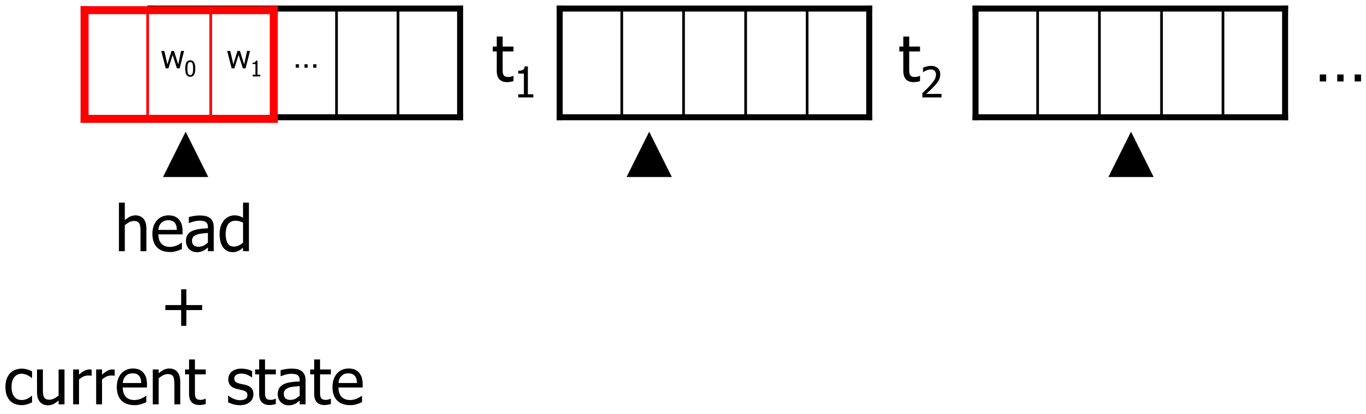

- game stores 3-cell window

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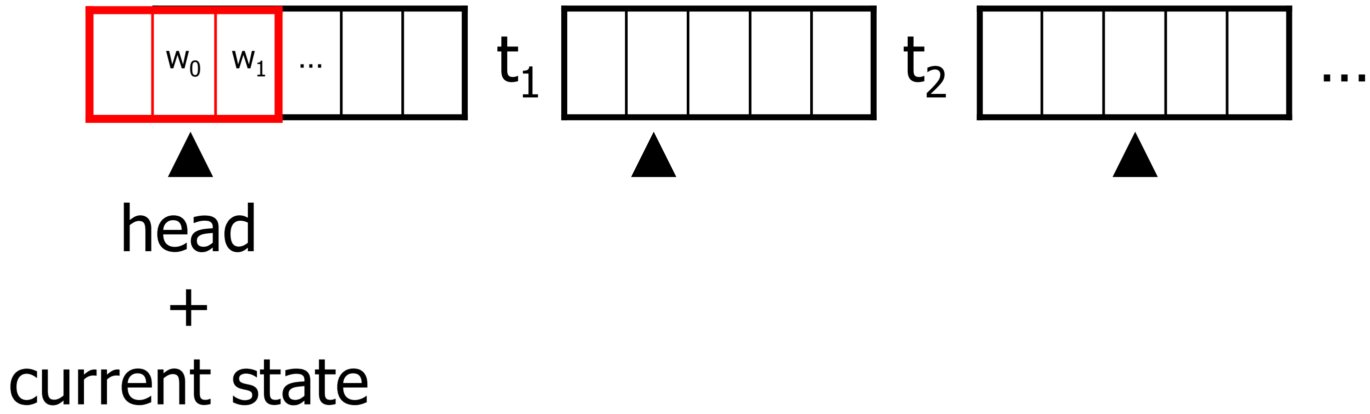

- game stores 3-cell window
- transition chosen by player 1 if ∃-state
  player 2 if ∀-state

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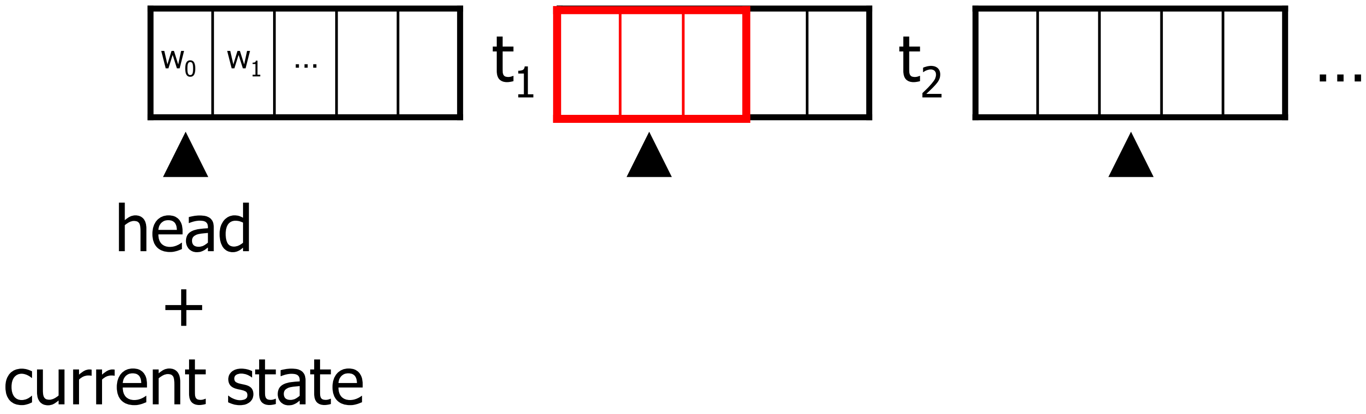

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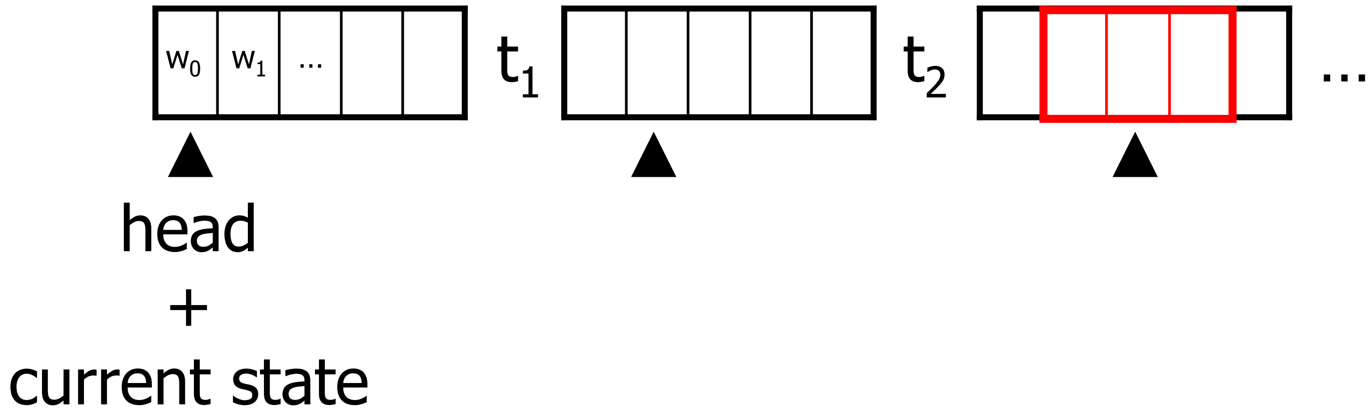

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Game graph:

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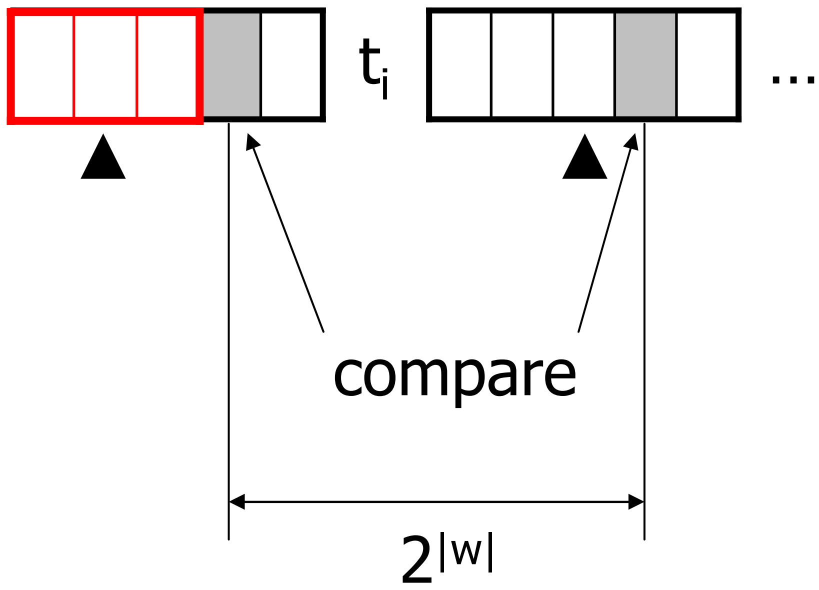

2EXPTIME-hardness

Three-player (sequential syn.): $\exists\sigma_1 \cdot \forall\sigma_2 \cdot \exists\sigma_3$

Game graph:

- initialization
- player 1 spells out a run of M on w:
- player 2 can check player 1's announcement
- player 3 checks distance is $2^{|w|}$

2EXPTIME-hardness

Three-player (sequential syn.): $\exists\sigma_1 \cdot \forall\sigma_2 \cdot \exists\sigma_3$

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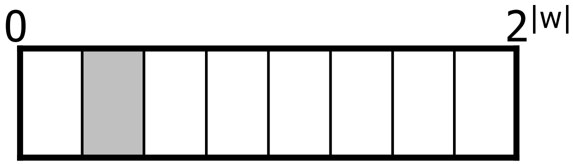
player 1



player 2

player 3

2EXPTIME-hardness

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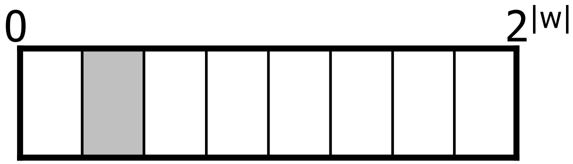
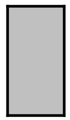
player 1 

player 2 0…000 (|w| bits) + stores 

player 3

2EXPTIME-hardness

Three-player (sequential syn.): $\exists\sigma_1 \cdot \forall\sigma_2 \cdot \exists\sigma_3$

Game graph:
- initialization
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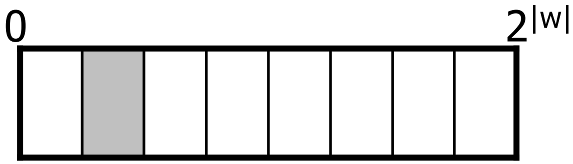
player 1



player 2 0…000 (|w| bits)

player 3 secretly stores one bit: 0

2EXPTIME-hardness

Three-player (sequential syn.): $\exists \sigma_1 \cdot \forall \sigma_2 \cdot \exists \sigma_3$

Game graph:

- initialization
- player 1 spells out a run of M on w:
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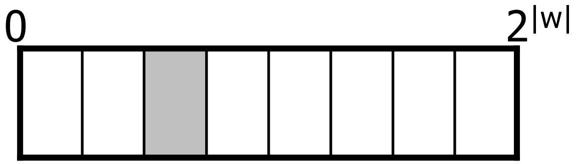
player 1



player 2 0…001 (|w| bits)

player 3 secretly stores one bit: 0

2EXPTIME-hardness

Three-player (sequential syn.): $\exists\sigma_1 \cdot \forall\sigma_2 \cdot \exists\sigma_3$

Game graph:
- initialization
- player 1 spells out a run of M on w:
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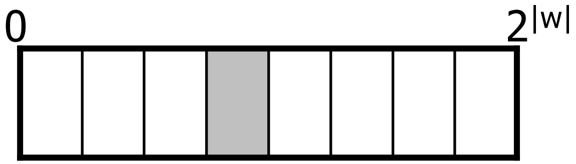
player 1



player 2 0…010 (|w| bits)

player 3 secretly stores one bit: 0

2EXPTIME-hardness

Three-player (sequential syn.): $\exists\sigma_1 \cdot \forall\sigma_2 \cdot \exists\sigma_3$

Game graph:
- initialization
- player 1 spells out a run of M on w:
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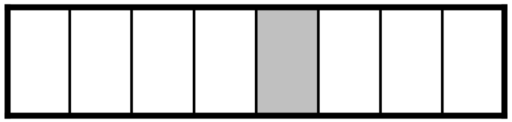
- player 3 checks distance is $2^{|w|}$

player 1 

player 2 0…011 (|w| bits)

player 3 secretly stores one bit: 0

2EXPTIME-hardness

Three-player (sequential syn.): $\exists\sigma_1 \cdot \forall\sigma_2 \cdot \exists\sigma_3$

Game graph:
- initialization
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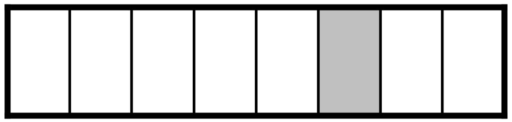
player 1 

player 2 0…100 (|w| bits)

player 3

secretly stores one bit: 1

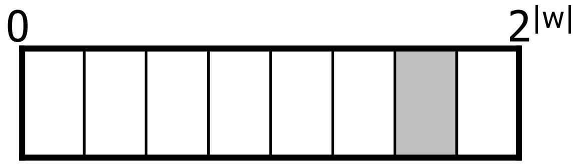
Updated when followed by 1^*

2EXPTIME-hardness

Three-player (sequential syn.): $\exists\sigma_1 \cdot \forall\sigma_2 \cdot \exists\sigma_3$

Game graph:
- initialization
- player 1 spells out a run of M on w:
- player 2 can check player 1's announcement
- player 3 checks distance is $2^{|w|}$

player 1



player 2 0…101 (|w| bits)

player 3 secretly stores one bit: 1

2EXPTIME-hardness

Three-player (sequential syn.): $\exists\sigma_1 \cdot \forall\sigma_2 \cdot \exists\sigma_3$

Game graph:

- initialization
- player 1 spells out a run of M on w:
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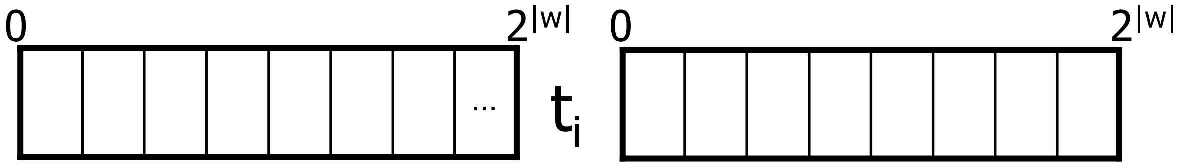
player 1



player 2 0…101 (|w| bits)

player 3 secretly stores one bit: 1

2EXPTIME-hardness

Three-player (sequential syn.): $\exists\sigma_1 \cdot \forall\sigma_2 \cdot \exists\sigma_3$

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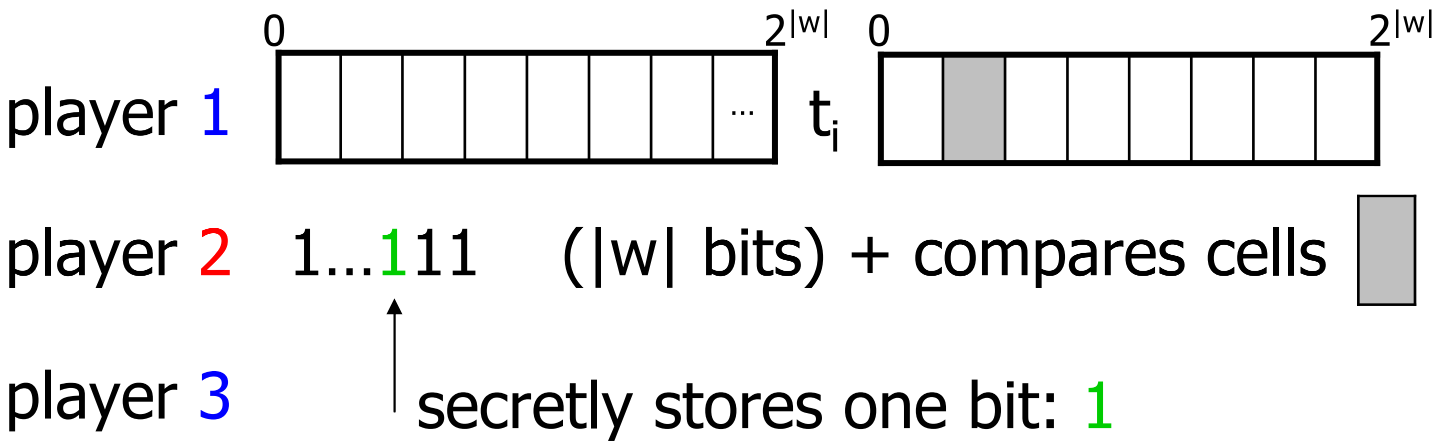

2EXPTIME-hardness

Three-player (sequential syn.): $\exists\sigma_1 \cdot \forall\sigma_2 \cdot \exists\sigma_3$

Game graph:
- initialization
- player 1 spells out a run of M on w:
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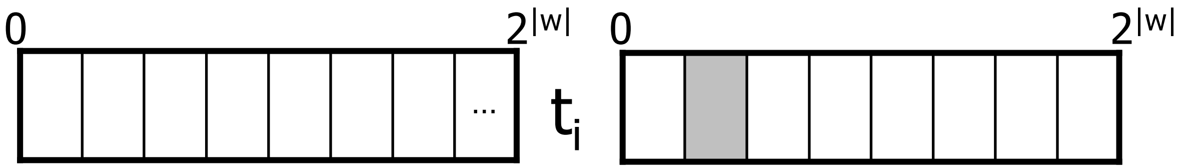
player 1



player 2 1…1 11 (|w| bits) + compares cells

player 3 secretly stores one bit: 1

2EXPTIME-c

Weak adversary

Three-player games:

$\exists \sigma_1 \cdot \forall \sigma_2 \cdot \exists \sigma_3$ player 1 **less** informed than player 2

2EXPTIME-c

Weak adversary

Three-player games:

$\exists\sigma_1 \cdot \forall\sigma_2 \cdot \exists\sigma_3$	player 1 **less** informed than player 2	2EXPTIME-c
$\exists\sigma_1 \cdot \forall\sigma_2 \cdot \exists\sigma_3$	player 1 **more** informed than player 2	?

when player 1 is perfect: non-elementary memory required for reachability

Weak adversary

Three-player games:

$\exists \sigma_1 \cdot \forall \sigma_2 \cdot \exists \sigma_3$	player 1 **less** informed than player 2	2EXPTIME-c
$\exists \sigma_1 \cdot \forall \sigma_2 \cdot \exists \sigma_3$	player 1 **more** informed than player 2	? (decidable when player 1 is perfect, for reachability and safety)

when player 1 is perfect: non-elementary memory required

(as in two-player stochastic games
with positive reachability objective)

Partial-Observation Games

Muti-player games (player 1 less informed)

$\exists \sigma_1 \cdot \forall \sigma_2 \cdot \exists \sigma_3$

2EXPTIME-c

Four-player Games

Muti-player games (player 1 less informed)

$\exists\sigma_1 \cdot \forall\sigma_2 \cdot \exists\sigma_3$

2EXPTIME-c

$\underbrace{\exists\sigma_1 \cdot \forall\sigma_2}_{\text{partial}} \cdot \underbrace{\exists\sigma_3 \cdot \forall\sigma_4}_{\text{perfect}}$

$\downarrow$

$\exists\sigma_1 \cdot \forall\sigma_2 \cdot \forall\sigma_4 \cdot \exists\sigma_3$

Four-player Games

Muti-player games (player 1 less informed)

$\exists\sigma_1 \cdot \forall\sigma_2 \cdot \exists\sigma_3$

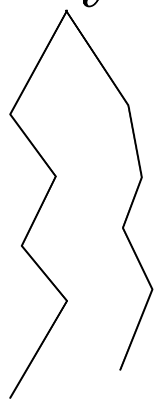
2EXPTIME-c

$\underbrace{\exists\sigma_1 \cdot \forall\sigma_2}_{\text{partial}} \cdot \underbrace{\exists\sigma_3 \cdot \forall\sigma_4}_{\text{perfect}}$

↓

$\underbrace{\exists\sigma_1 \cdot \forall\sigma_2}_{t^{\sigma_1,\sigma_2}} \cdot \underbrace{\forall\sigma_4 \cdot \exists\sigma_3}_{\text{Alternating tree automaton}}$

t^{σ_1,σ_2}



branching: $\mathcal{O}_2$

label: $A_1 \times A_2$

Four-player Games

Muti-player games (player 1 less informed)

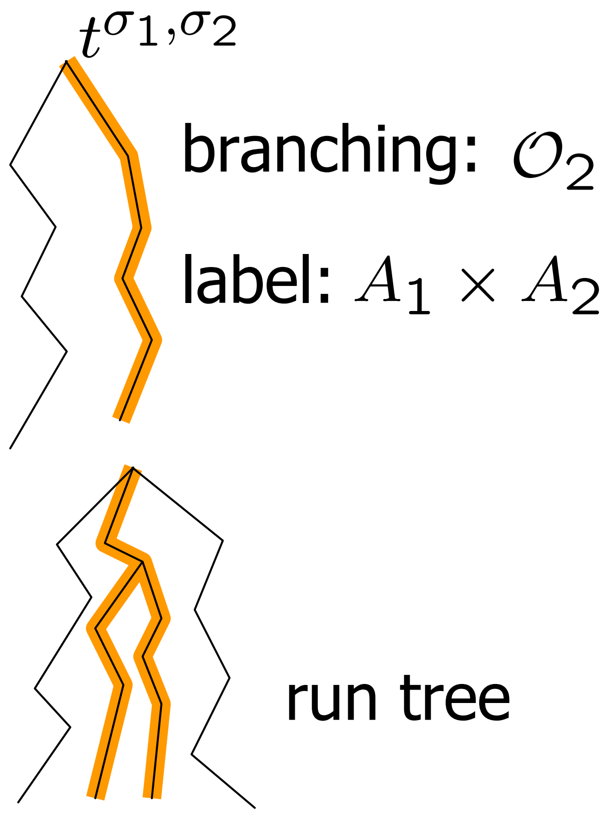
$\exists\sigma_1 \cdot \forall\sigma_2 \cdot \exists\sigma_3$

2EXPTIME-c

$\underbrace{\exists\sigma_1 \cdot \forall\sigma_2}_{\text{partial}} \cdot \underbrace{\exists\sigma_3 \cdot \forall\sigma_4}_{\text{perfect}}$

↓

$\underbrace{\exists\sigma_1 \cdot \forall\sigma_2}_{t^{\sigma_1,\sigma_2}} \cdot \underbrace{\forall\sigma_4 \cdot \exists\sigma_3}_{\text{Alternating tree automaton}}$



Four-player Games

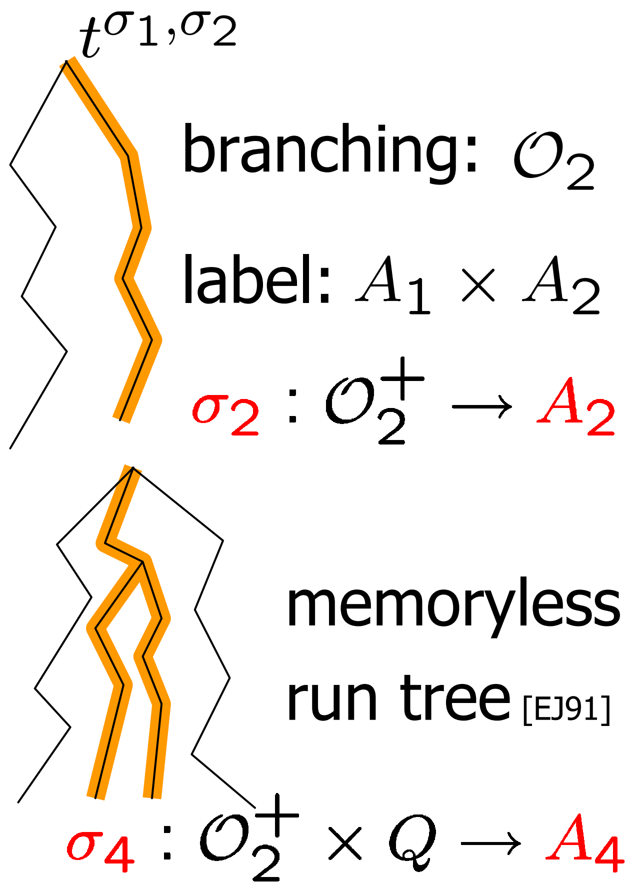
Muti-player games (player 1 less informed)

$\exists\sigma_1 \cdot \forall\sigma_2 \cdot \exists\sigma_3$

2EXPTIME-c

$\underbrace{\exists\sigma_1 \cdot \forall\sigma_2}_{\text{partial}} \cdot \underbrace{\exists\sigma_3 \cdot \forall\sigma_4}_{\text{perfect}}$

$\underbrace{\exists\sigma_1 \cdot \forall\sigma_2}_{t^{\sigma_1,\sigma_2}} \cdot \underbrace{\forall\sigma_4 \cdot \exists\sigma_3}_{\text{Alternating tree automaton}}$



Four-player Games

Muti-player games (player 1 less informed)

$\exists\sigma_1 \cdot \forall\sigma_2 \cdot \exists\sigma_3$

2EXPTIME-c

$\exists\sigma_1 \cdot \forall\sigma_2 \cdot \exists\sigma_3 \cdot \forall\sigma_4$

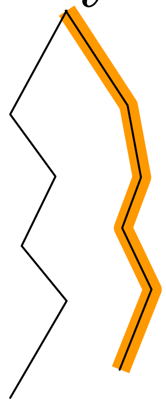
partial | perfect

$\exists\sigma_1 \cdot \forall\sigma_2 \cdot \forall\sigma_4 \cdot \exists\sigma_3$

$\exists\sigma_1 \cdot \forall\sigma_{24} \cdot \exists\sigma_3$

$\sigma_{24} : \mathcal{O}_2^+ \to A_2 \times (Q \to A_4)$

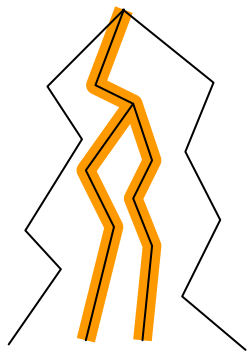
t^{σ_1,σ_2}



branching: $\mathcal{O}_2$

label: $A_1 \times A_2$

$\sigma_2 : \mathcal{O}_2^+ \to A_2$



memoryless

run tree [EJ91]

$\sigma_4 : \mathcal{O}_2^+ \times Q \to A_4$

Four-player Games

Muti-player games (player 1 less informed)

$\exists\sigma_1 \cdot \forall\sigma_2 \cdot \exists\sigma_3$ 2EXPTIME-c

$\exists\sigma_1 \cdot \forall\sigma_2 \cdot \exists\sigma_3 \cdot \forall\sigma_4$ 2EXPTIME-c

partial | perfect

↓

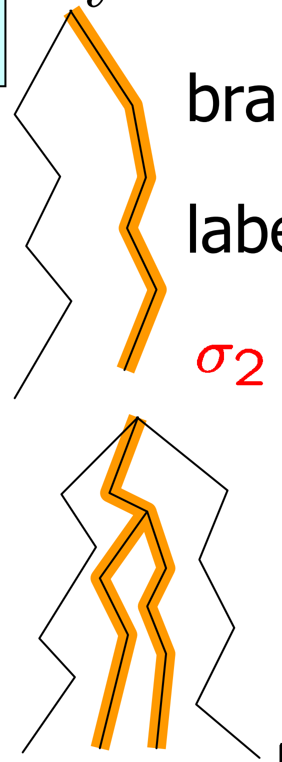
$\exists\sigma_1 \cdot \forall\sigma_2 \cdot \forall\sigma_4 \cdot \exists\sigma_3$

↓

$\exists\sigma_1 \cdot \forall\sigma_{24} \cdot \exists\sigma_3$

$\sigma_{24} : \mathcal{O}_2^+ \to A_2 \times (Q \to A_4)$

t^{σ_1,σ_2}



branching: $\mathcal{O}_2$

label: $A_1 \times A_2$

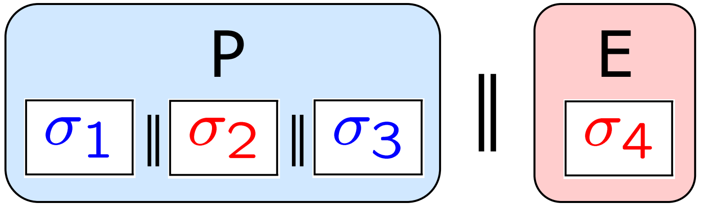
$\sigma_2 : \mathcal{O}_2^+ \to A_2$

memoryless run tree [EJ91]

$\sigma_4 : \mathcal{O}_2^+ \times Q \to A_4$

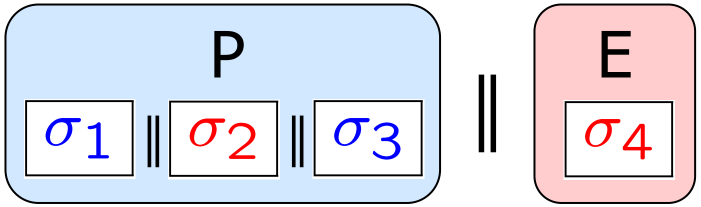
Applications

- Sequential synthesis



Applications

- Sequential synthesis



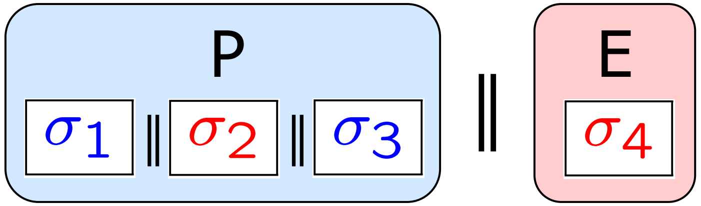
- Stochastic games with finite-memory strategies
  (and parity objective)

Probabilistic states "simulated" by two-player gadget (for positive and almost-sure winning)

Two-player stochastic game reduces to four-player det. game

Applications

- Sequential synthesis



- Stochastic games with finite-memory strategies
(and parity objective)

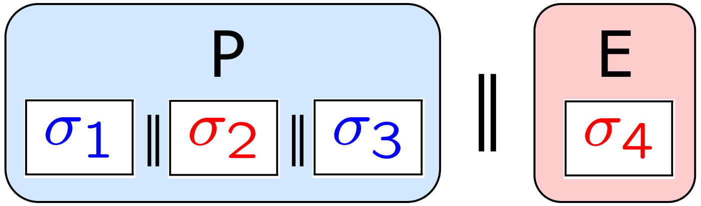
Probabilistic states "simulated" by two-player gadget (for positive and almost-sure winning)

Two-player stochastic game reduces to four-player det. game

Almost-sure and positive two-player stochastic parity games with finite memory and player 1 less informed is 2EXPTIME-c

Applications

- Sequential synthesis



- Stochastic games with finite-memory strategies (and parity objective)

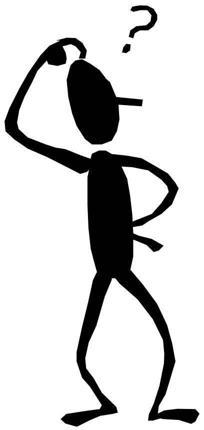
  - Probabilistic states "simulated" by two-player gadget (for positive and almost-sure winning)
  - Two-player stochastic game reduces to four-player det. game

- Multi-player games

$$\underbrace{\exists\sigma_1 \cdot \forall\sigma_2}_{\text{partial}} \cdot \underbrace{\exists\sigma_3 \cdot \forall\sigma_4}_{\text{perfect}}$$

The End

∀ Thank you !



∃ Questions ?