Toward a structure theory of ω-regular trace languages

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RWTH AACHEN UNIVERSITY

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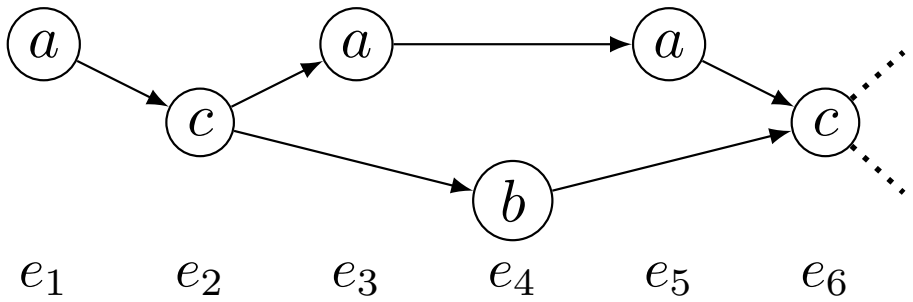
22 May 2014

traces and trace languages

Models of concurrent executions of distributed systems



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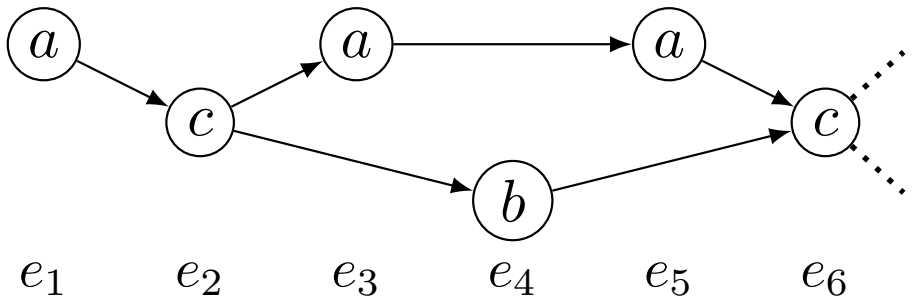

Automata recognizing languages of traces

- Finite asynchronous automata (FAA) for languages of finite traces
- Deterministic asynchronous Büchi automata (DABA) and deterministic asynchronous Muller automata (DAMA) for languages of infinite traces

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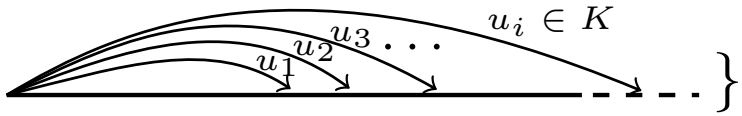
- $\lim(K) := \{\alpha \in \Sigma^\omega \mid \exists^\infty i \in \mathbb{N}, \alpha =$ 

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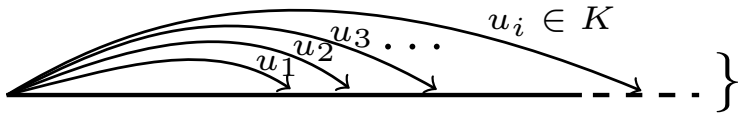
- $\lim(K) := \{\alpha \in \Sigma^\omega \mid \exists^\infty i \in \mathbb{N}, \alpha =$ 

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Theorem (det. Büchi automata)

If a DFA $\mathcal{A} = (Q, \Sigma, q_0, \delta, F)$ *recognizes the language* $K \subseteq \Sigma^*$ *then the DBA* $\mathfrak{A} = (Q, \Sigma, q_0, \delta, F)$ *recognizes* $\lim(K)$.

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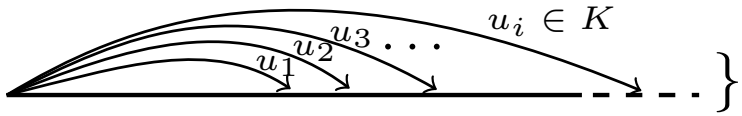
- $\lim(K) := \{\alpha \in \Sigma^\omega \mid \exists^\infty i \in \mathbb{N}, \alpha =$ 

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A language $L \subseteq \Sigma^\omega$ is DBA recognizable iff for each Muller automaton recognizing L the acceptance component is closed under supersets.

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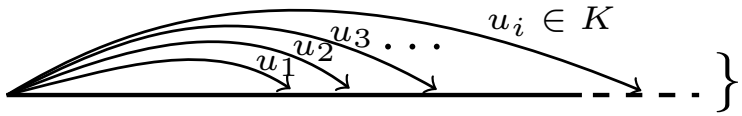
- $\lim(K) := \{\alpha \in \Sigma^\omega \mid \exists^\infty i \in \mathbb{N}, \alpha =$ 

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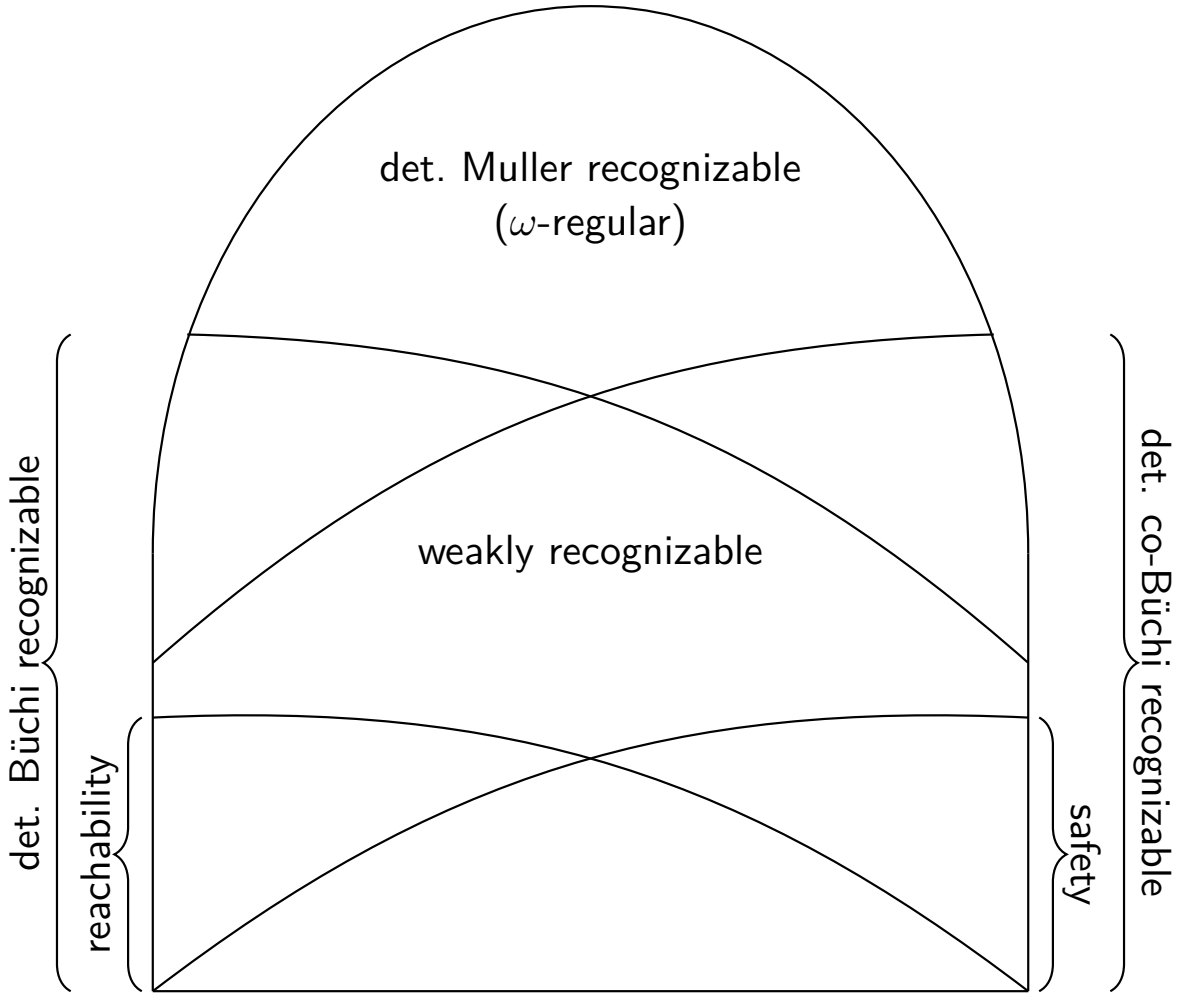
Theorem (R. Büchi, R. McNaughton)

A language $L \subseteq \Sigma^\omega$ is DMA recognizable (L is ω-regular) iff L can be expressed as a finite Boolean combination of DBA recognizable languages.

the Borel classification of ω-regular languages



fundamental questions

- FAA recognizing T to DABA recognizing "$\lim(T)$"?
- Relation between DABA and DAMA as in Landweber's theorem?
- Is the class of ω-regular trace languages same as the class of Boolean combinations of DABA recognizable languages?

preliminaries: finite and infinite traces

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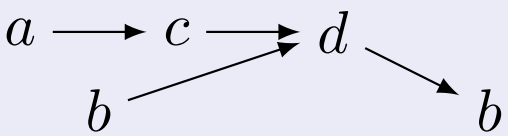

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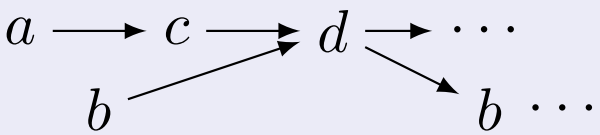

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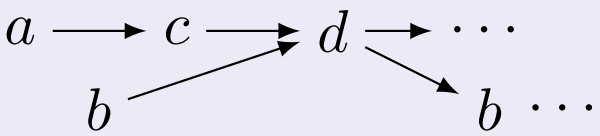

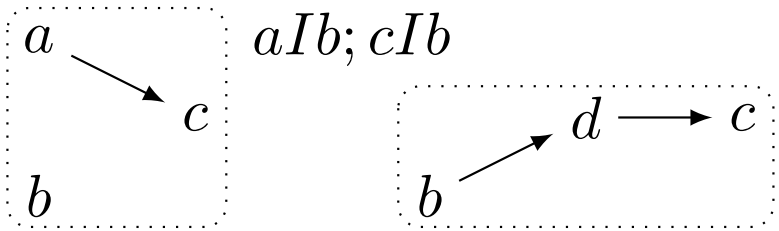

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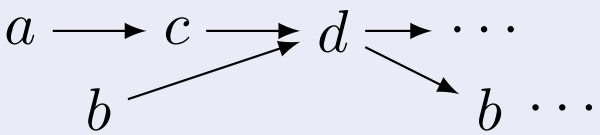

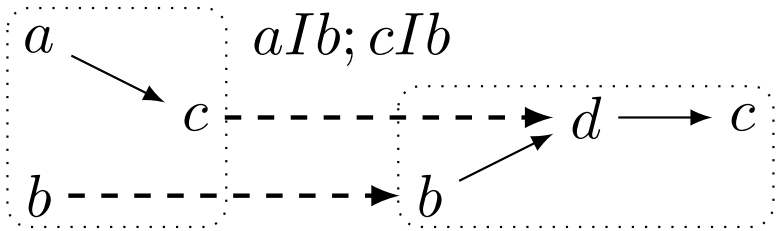

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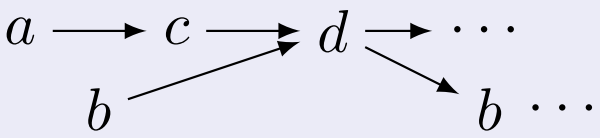

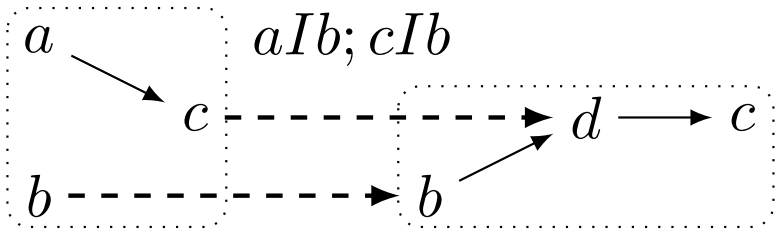

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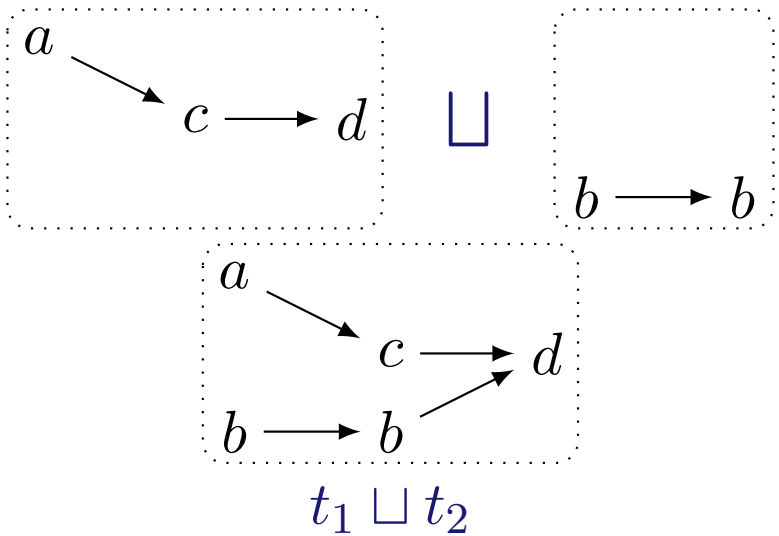

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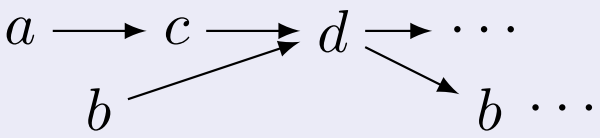

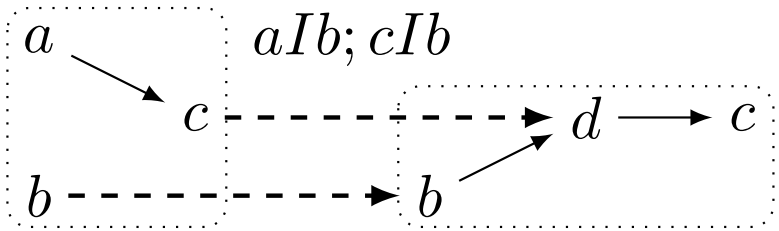

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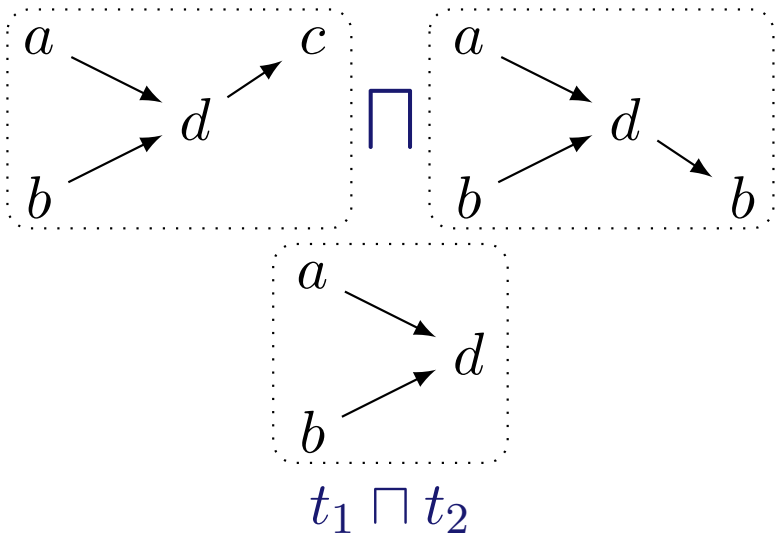

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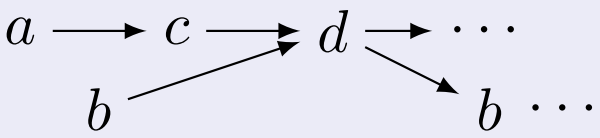
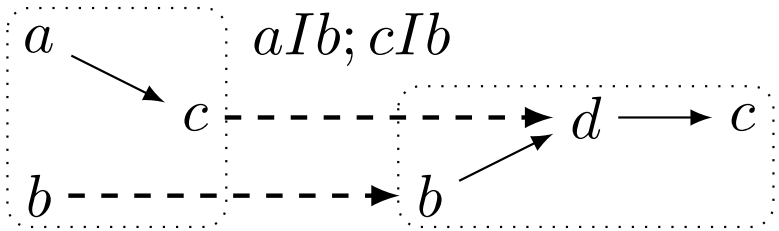


The set of all finite traces: $\mathbb{M}(\Sigma, I)$

The set of all infinite traces: $\mathbb{R}(\Sigma, I)$



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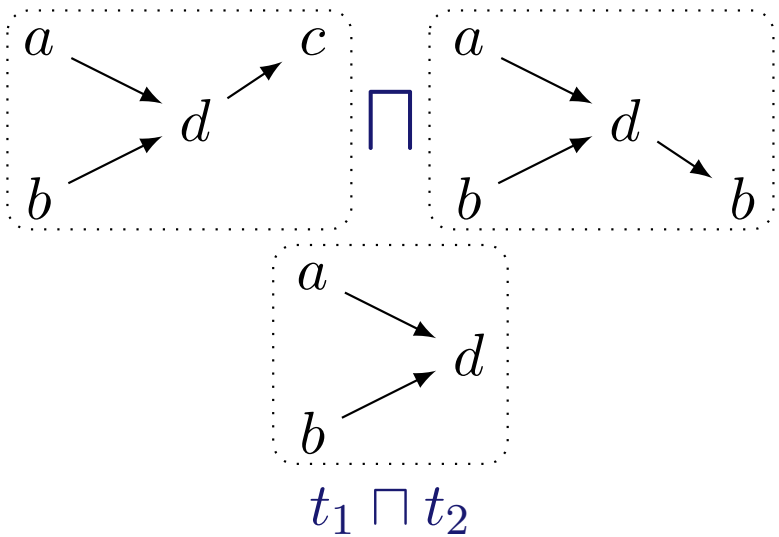

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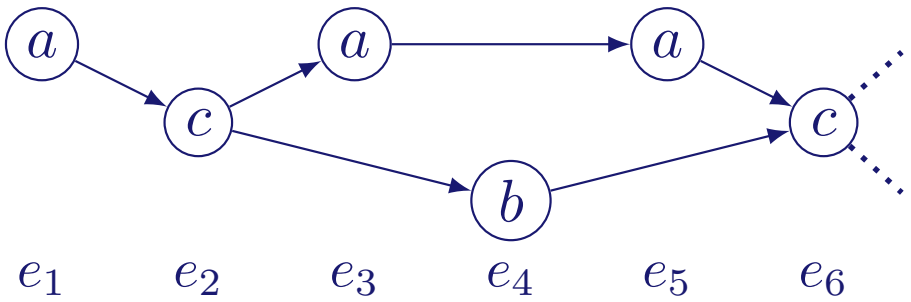

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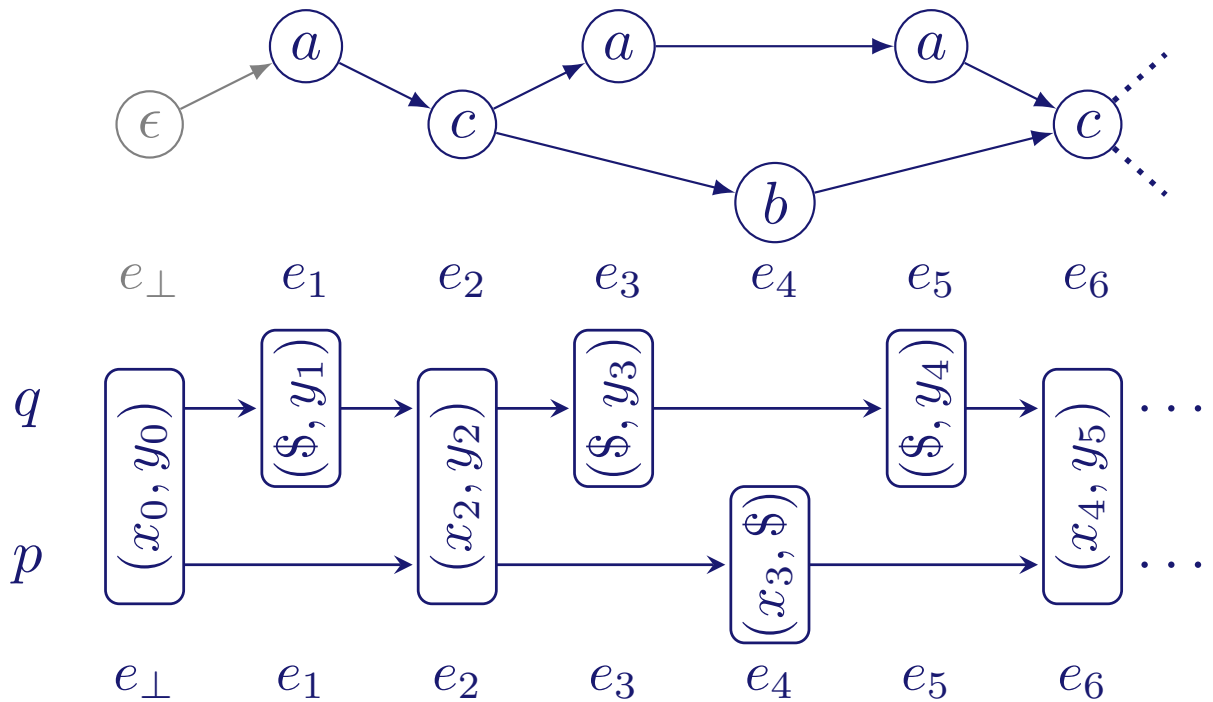

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Corresponding* run

$\rho = [V', (\lessdot_p)_{p \in \mathcal{P}}, \lambda', \Lambda]$

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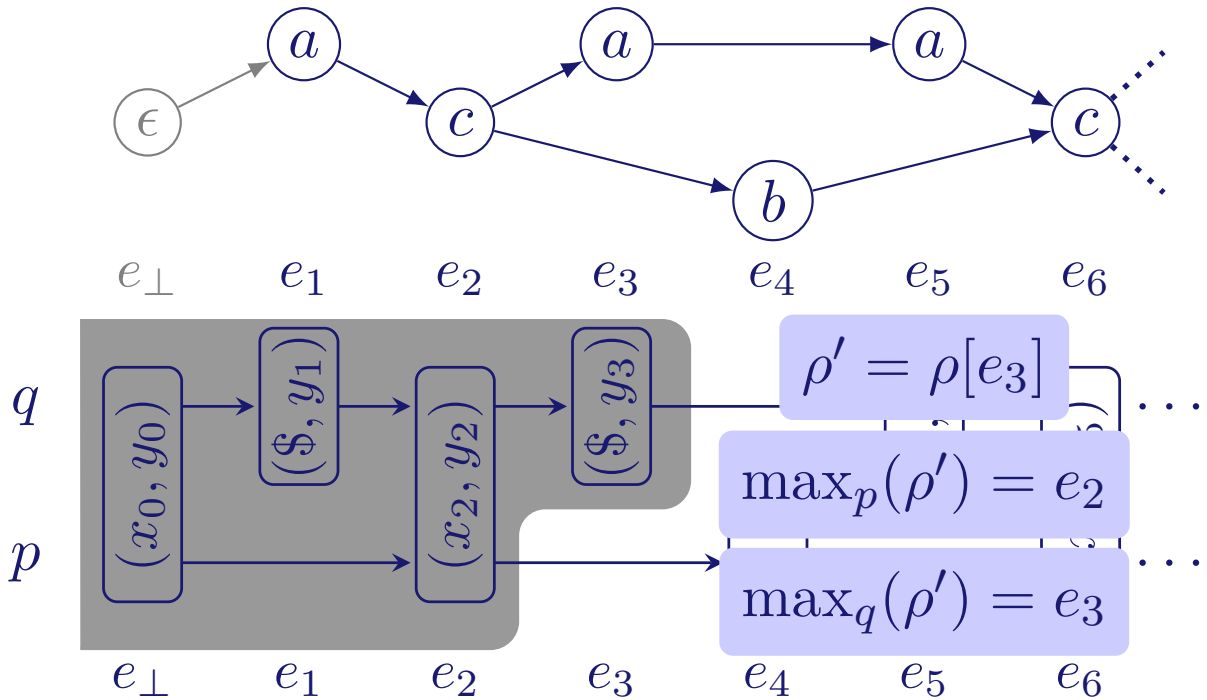

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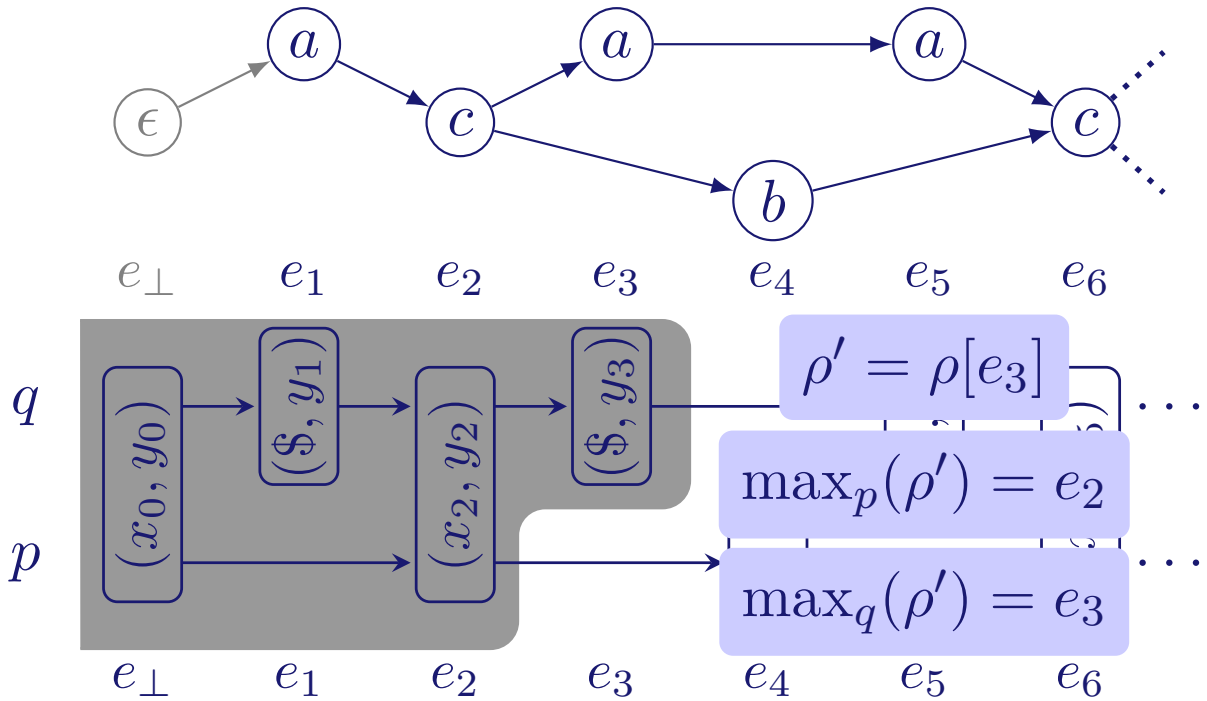

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Formally, given (Σ, I) and a mapping $\mathrm{dom}\colon \Sigma \to 2^{\mathcal{P}}$, an **asynchronous transition system** (an **ATS**) is a tuple $\mathfrak{T} = \big((X_p)_{p\in\mathcal{P}}, (\delta_a)_{a\in\Sigma}, \pi_0\big)$.

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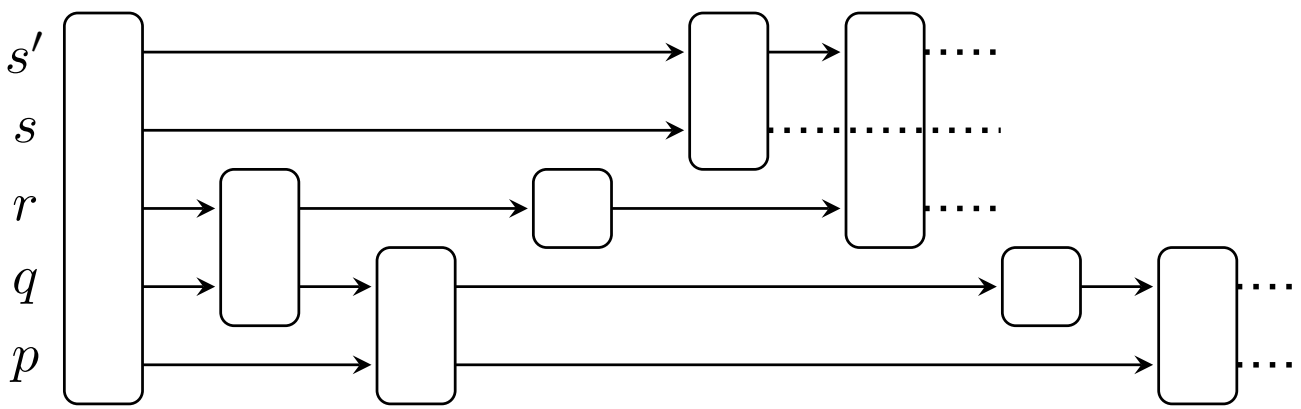

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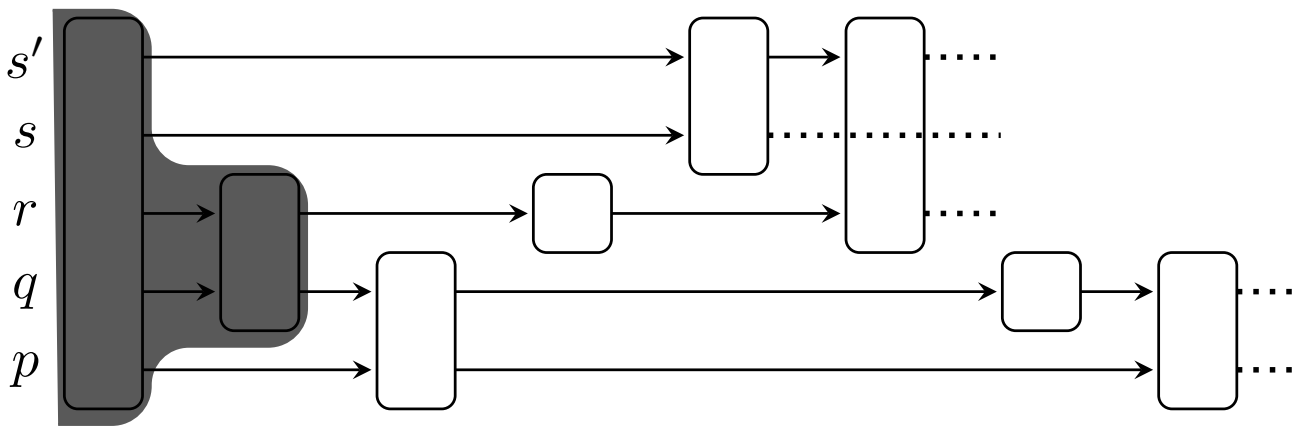

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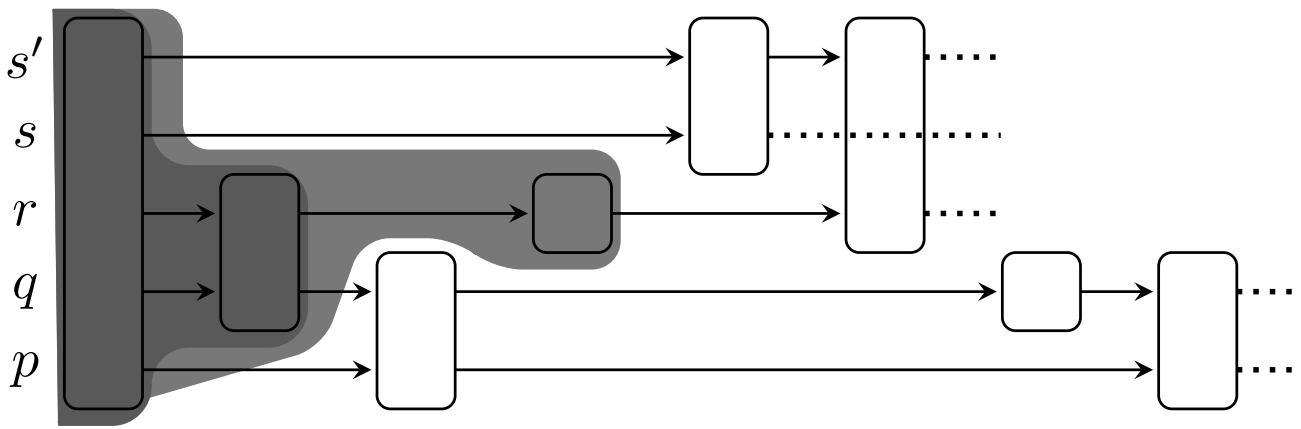

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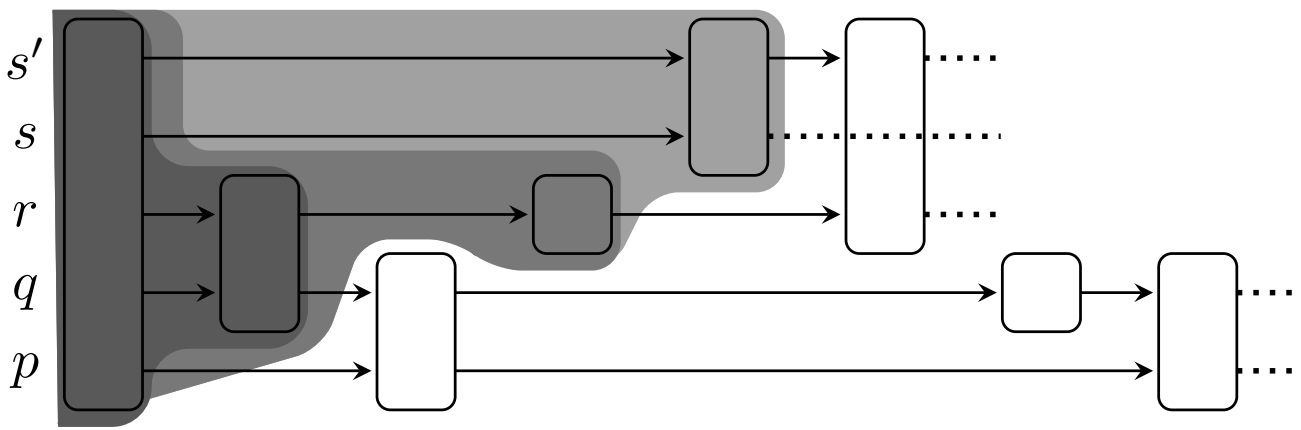

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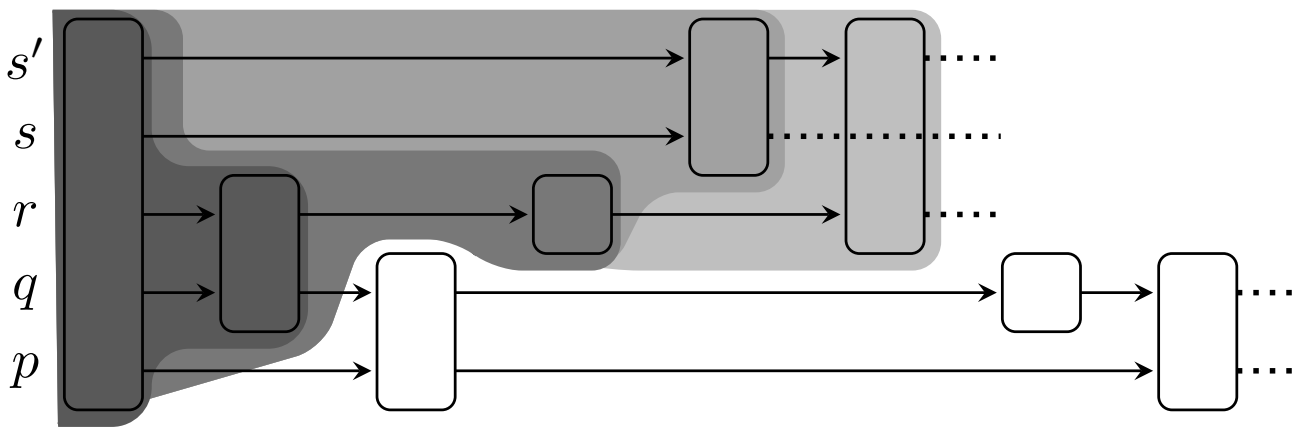

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A language $\Theta \in \mathbb{R}(\Sigma, I)$ is called an **ω-regular trace language** if there exists a DAMA recognizing it.

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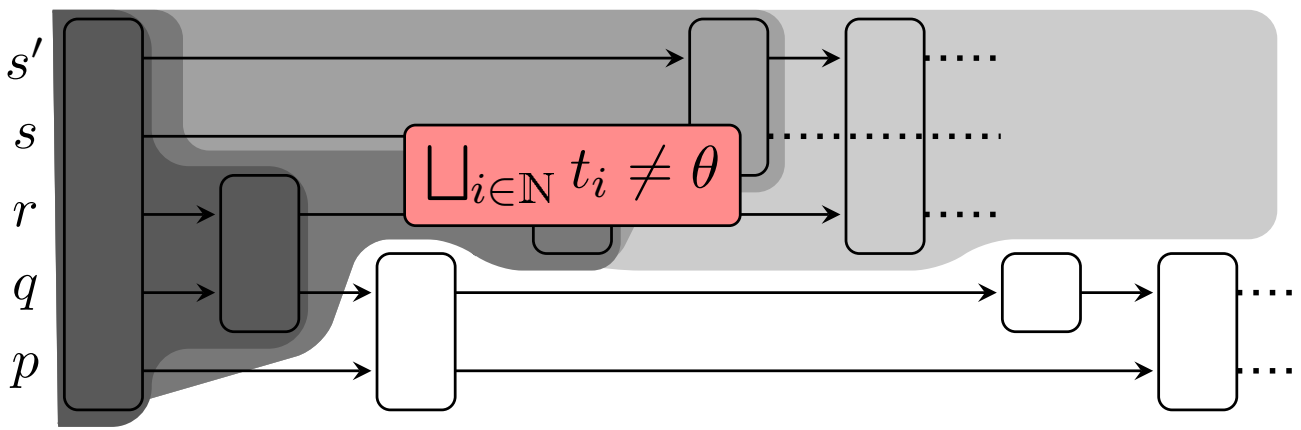

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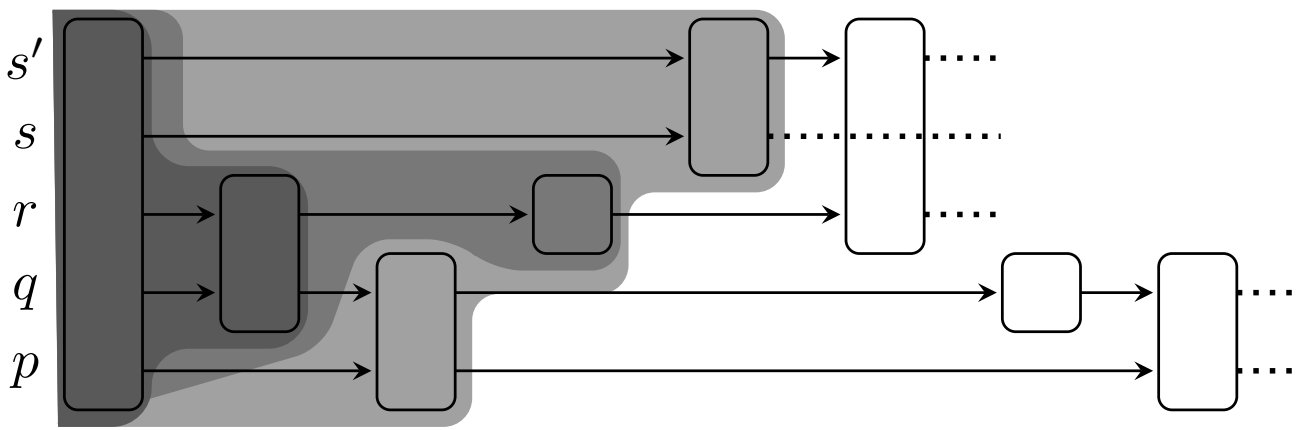

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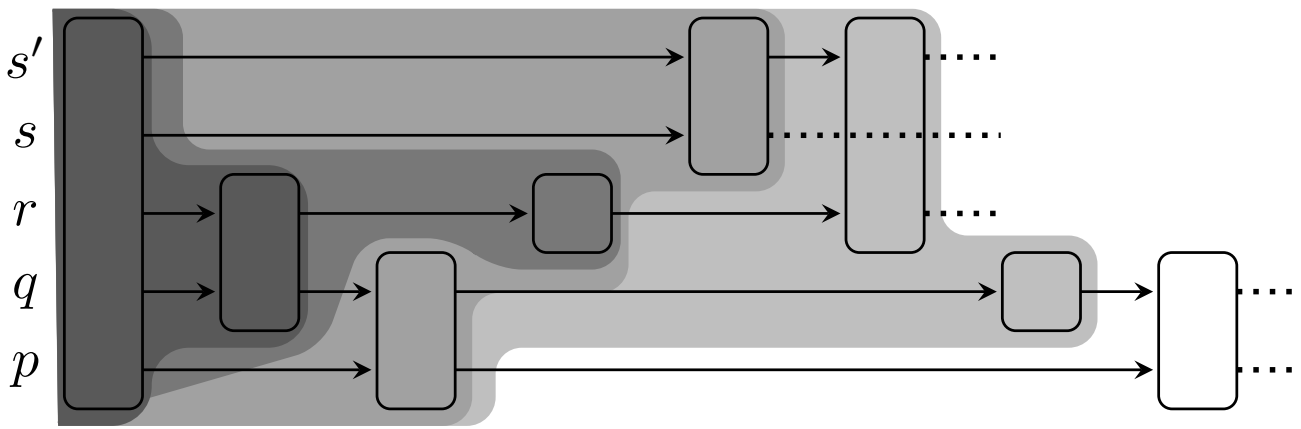

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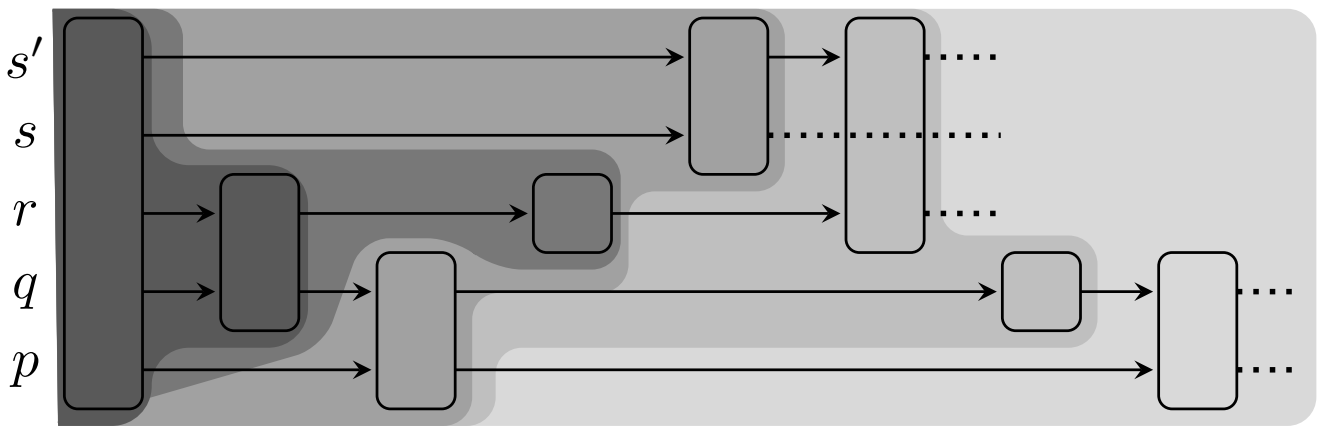

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A language $\Theta \subseteq \mathbb{R}(\Sigma, I)$ is called a **deterministic trace language** if Θ can be expressed as a finite union $\bigcup_i \lim_{A_i}(T_i)$ for regular trace languages $T_i \subseteq \mathbb{M}(\Sigma, I)$ and $A_i \subseteq \Sigma$.

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The challenge

The class of deterministic trace languages has no equivalent among the known classes of deterministic asynchronous automata.

motivation

State of fundamental questions

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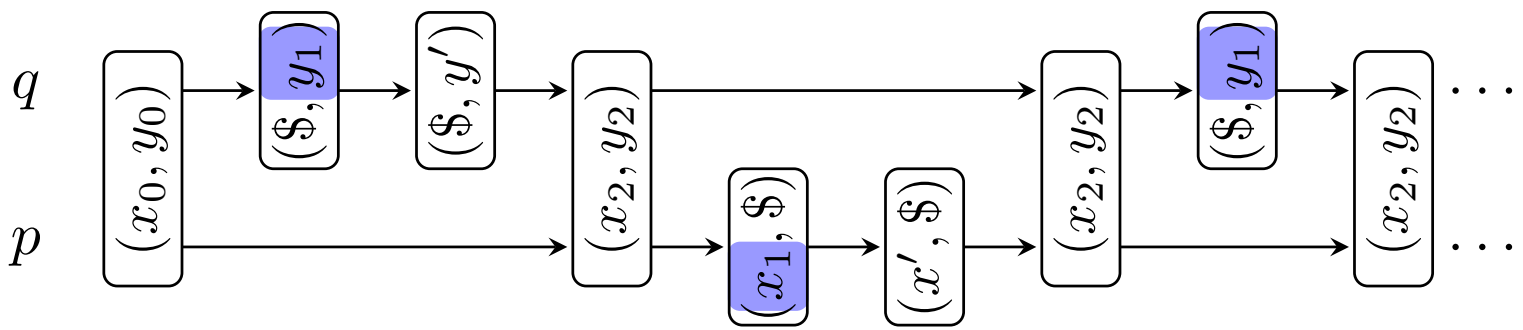

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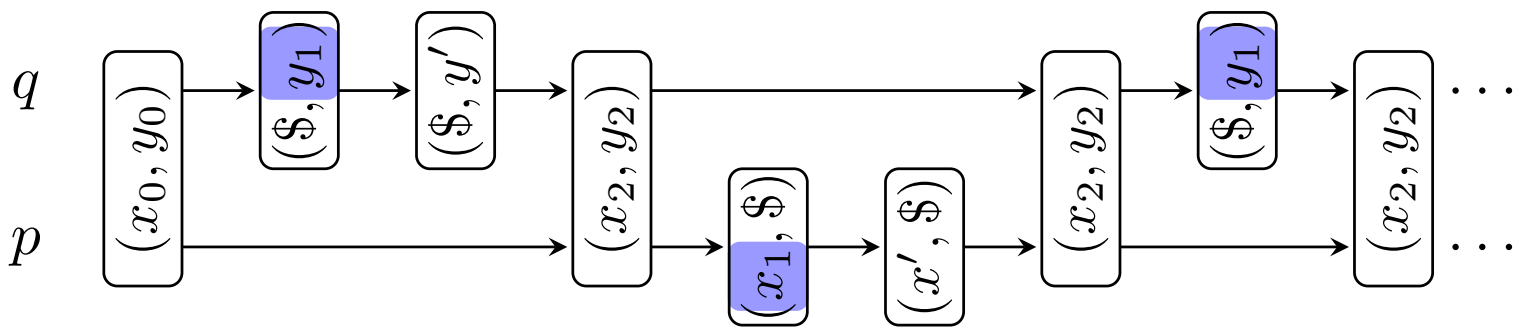

- DABA and DAMA vs. Landweber's characterization: **open**

- ω-regular trace languages (or DAMA recognizable languages) vs. Boolean combinations of DABA recognizable languages: **open**

summary of results

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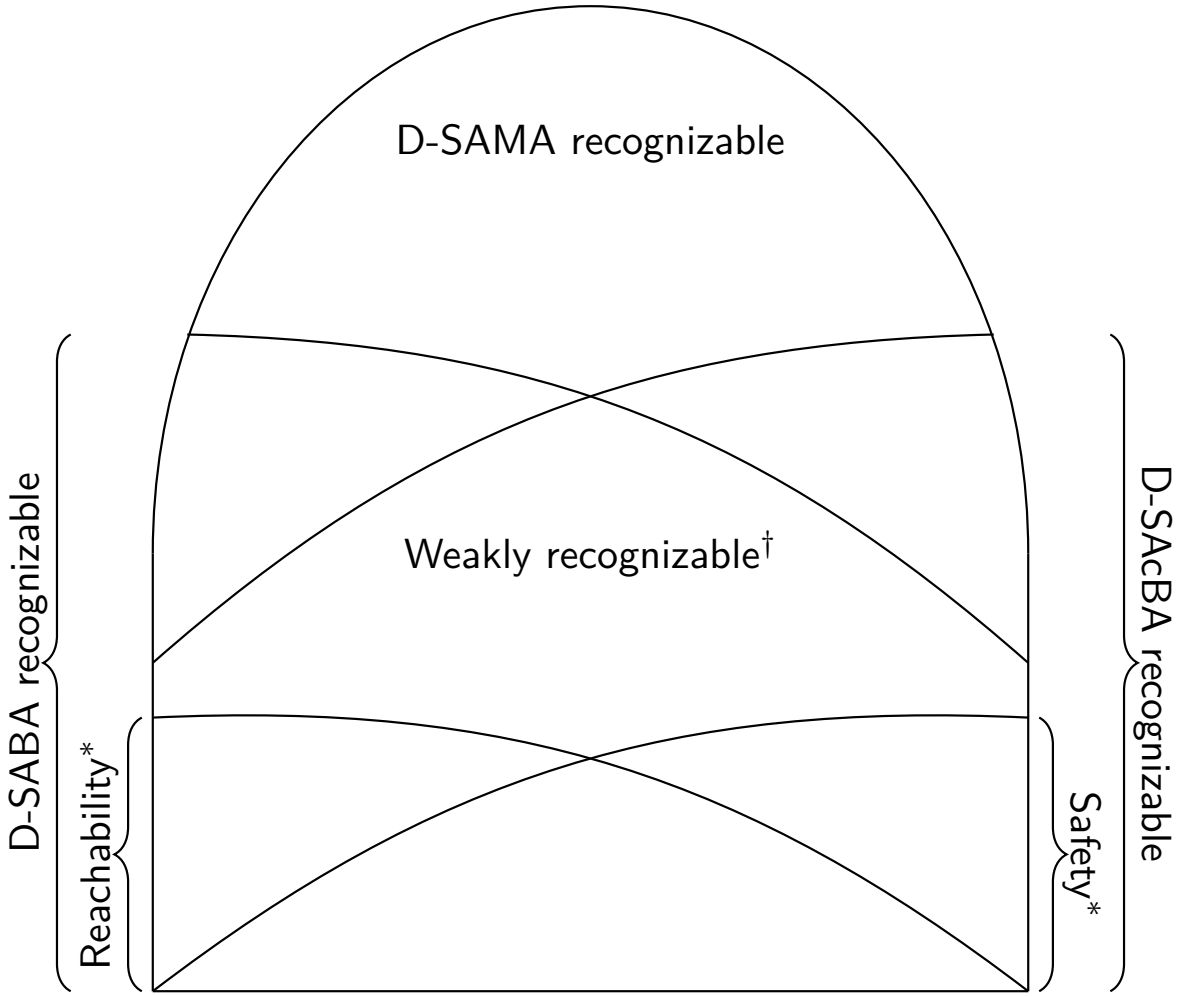
Work in progress:

- "Weak automata" – recognizing languages that are both det. Büchi and det. co-Büchi recognizable.
- Characterization of weak languages in terms of recognition by a special subset of Muller automata.

summary of results. . . in a picture



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* Impossible to characterize in terms of asynchronous automata

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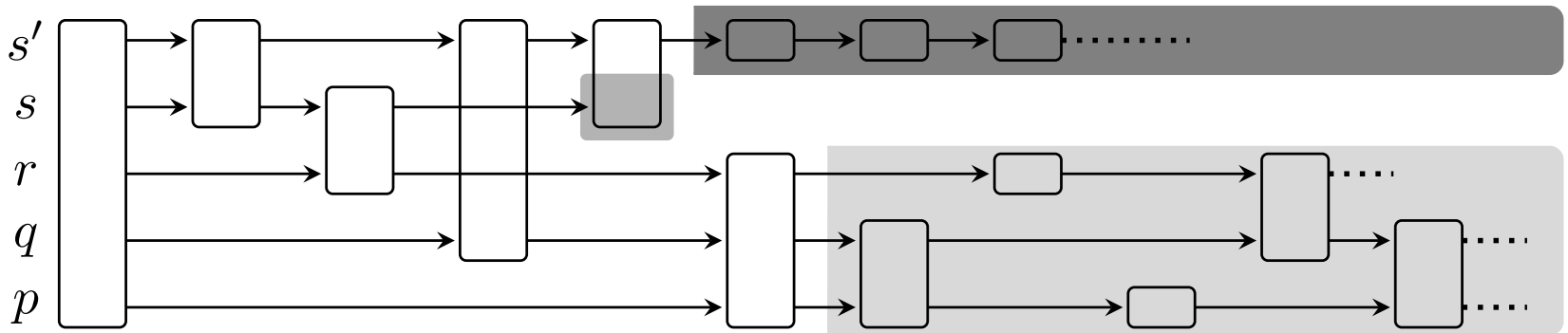

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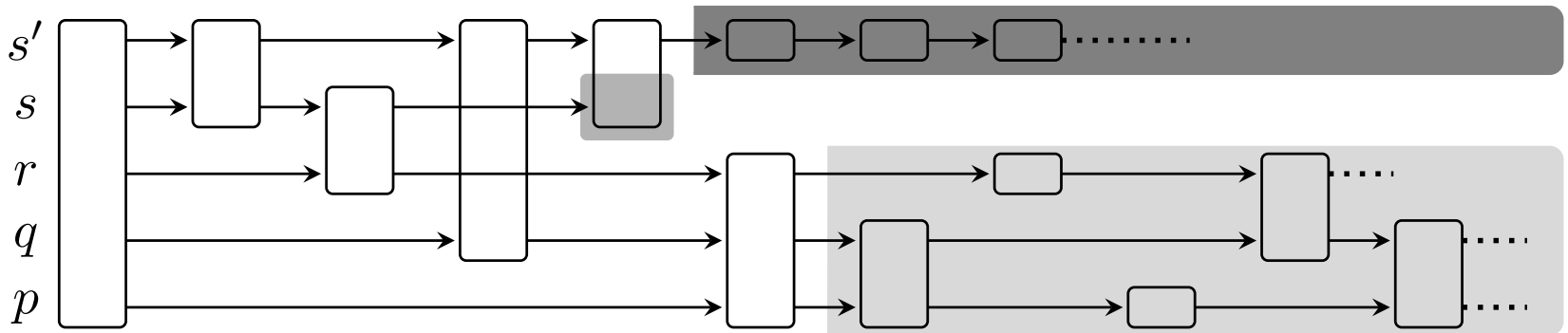

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How can each process $p \in \mathcal{P}$ infer the minimal part $P_i \in \Psi$ containing all the processes with whom it interacts infinitely often?

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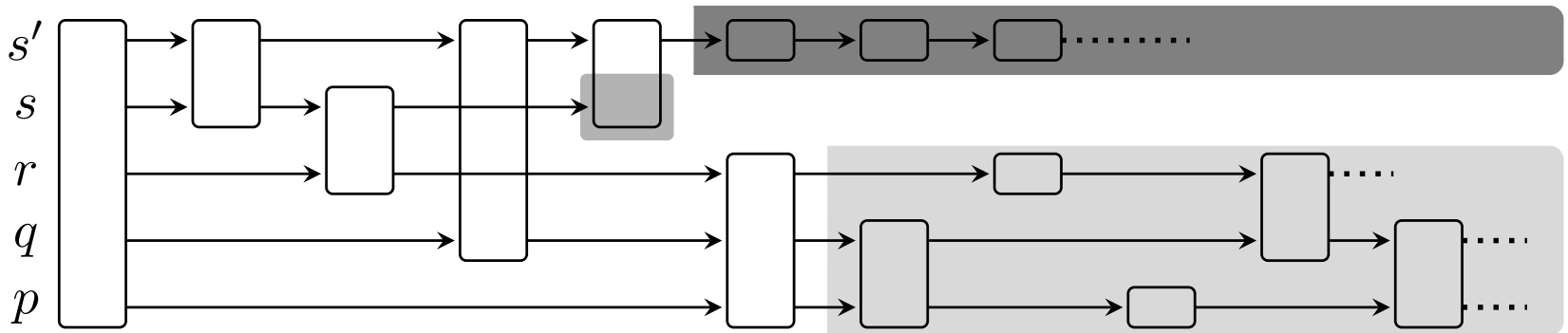

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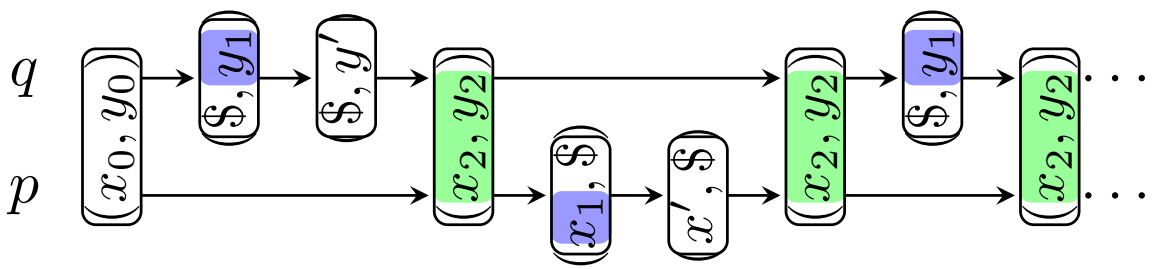

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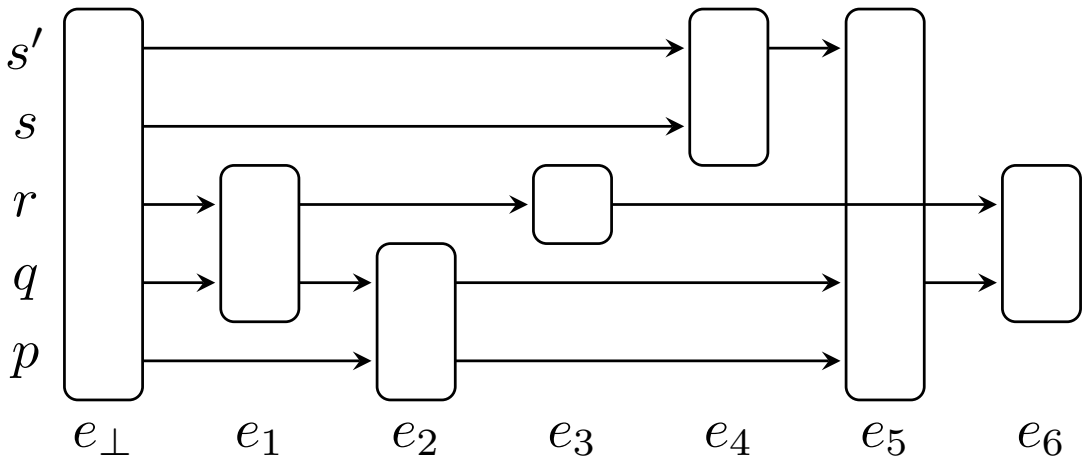

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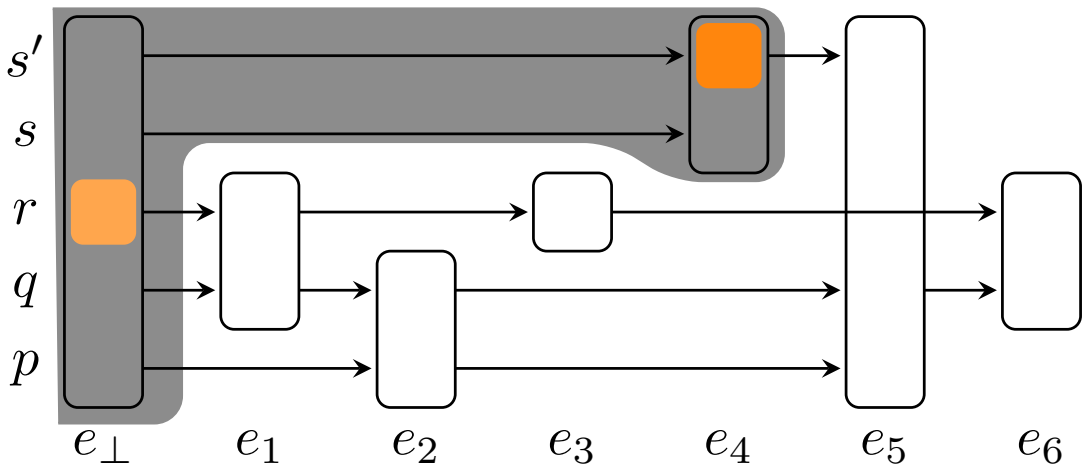

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$$\text{latest}_{s' \to r}(\rho') = e_\perp$$

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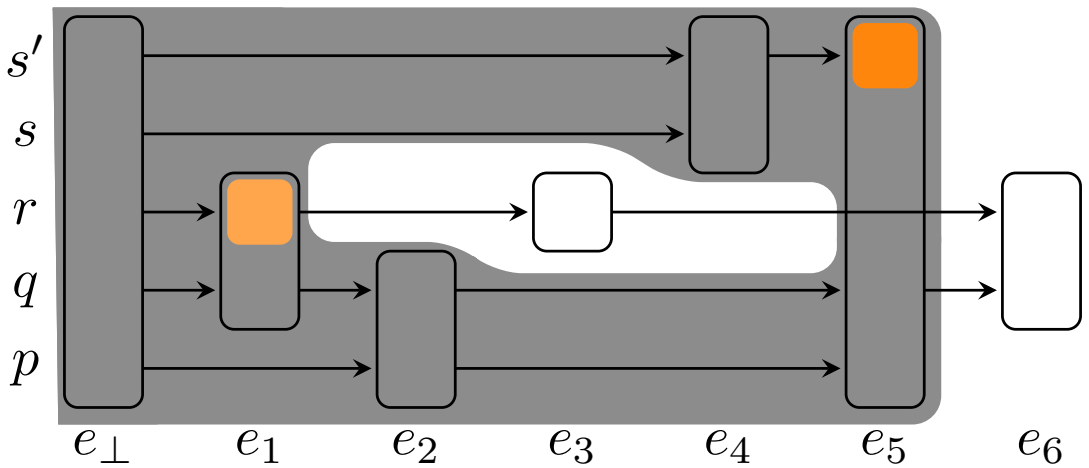

First hand information

$$\mathrm{latest}_{s' \to r}(\rho') = e_1$$

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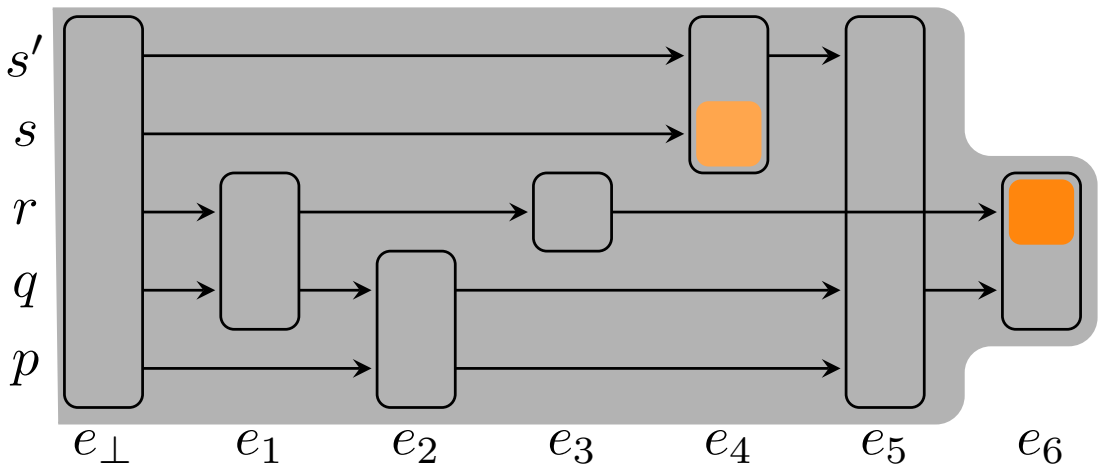
interactions and gossip



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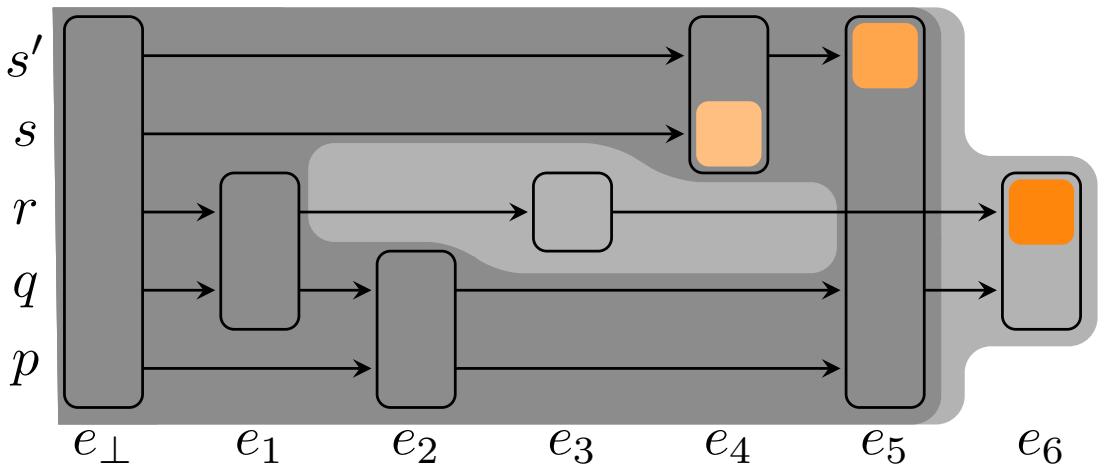

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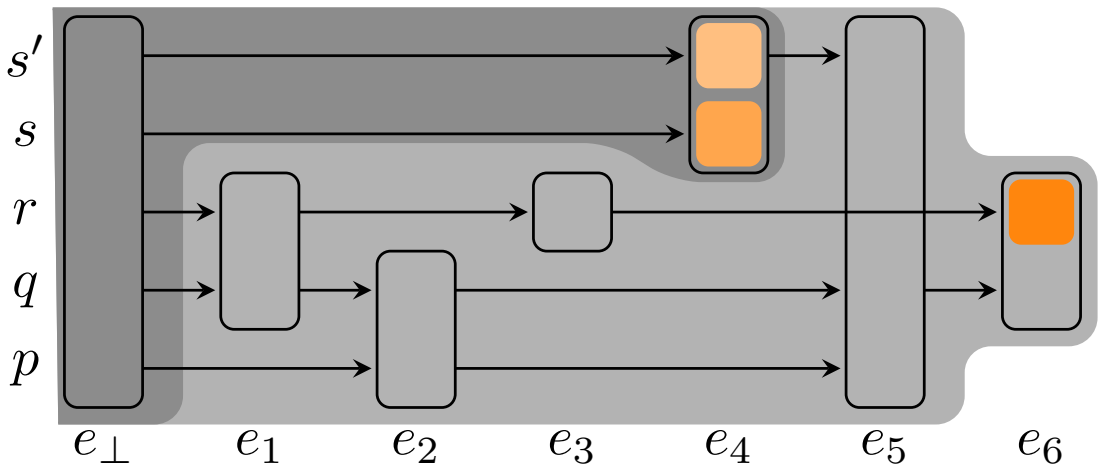

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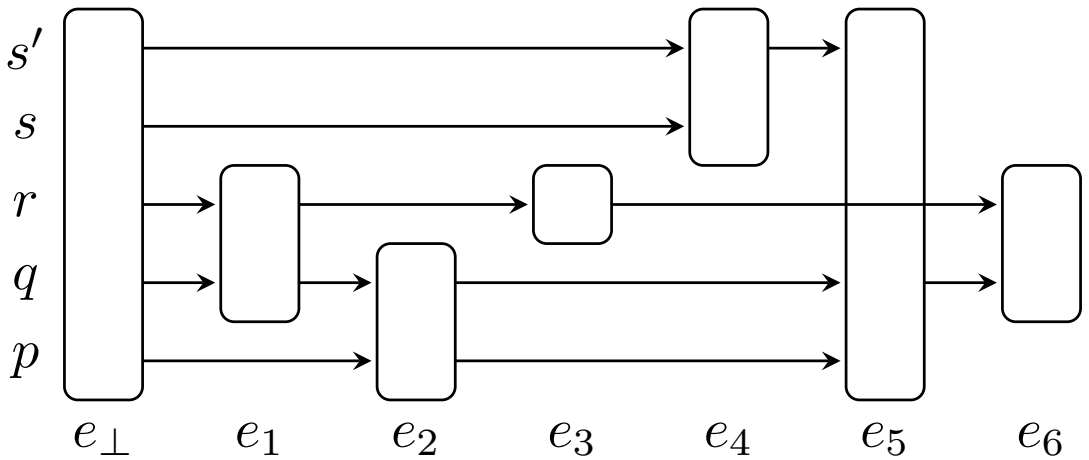

The **future** of e in ρ, $\mathrm{future}_\rho(e) := \{f \in \rho \mid f \in F_\rho \wedge e \leq f\}$.

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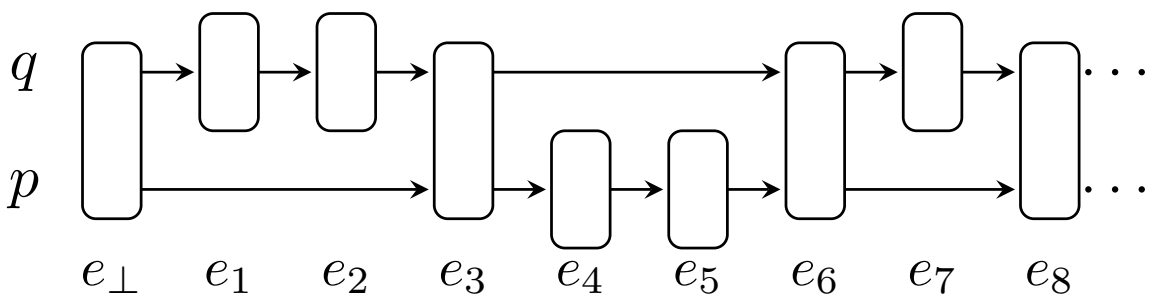

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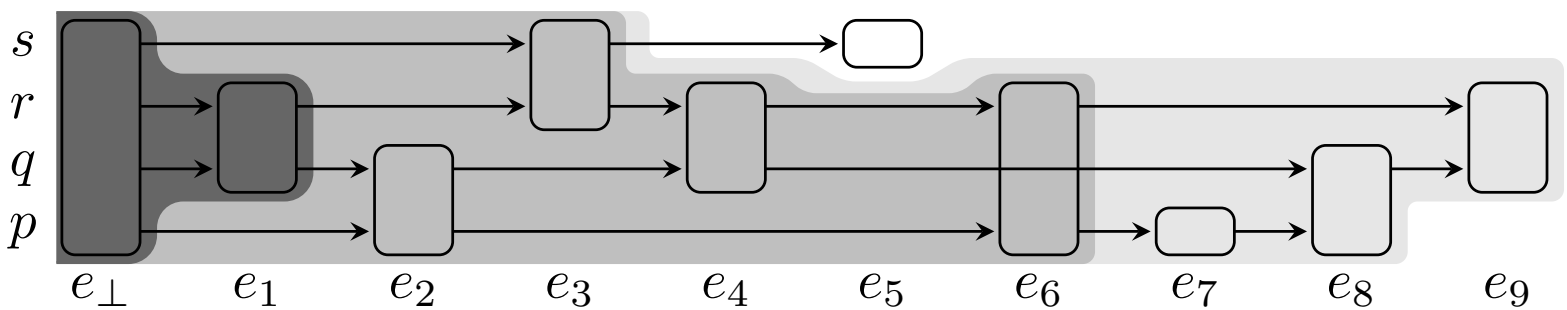

Example on board: compute $\text{ds}(e_8) = \mathcal{P}$ and $\text{ds}(e_9) = \{p, q, r\}$

degrees of synchronization: utility

Lemma (infinitary behavior)

Over an infinite run ρ, if a process p remains "live", then there exists a unique, maximal set $P \subseteq \mathcal{P}$ such that for infinitely many p-events $e \in \rho : \mathrm{ds}(e) = P$.

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Call the set P as mentioned above the **max-degree of p-synchronizations in ρ**, denoted by $\lceil \mathrm{ds}_p(\rho) \rceil$.
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If a run ρ induces a partition Ψ of the set of states, then for each part $P_i \in \Psi : q \in P_i \Leftrightarrow \lceil \mathrm{ds}_q(\rho) \rceil = P_i$.

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A set $X \subseteq X_p$ of local p-states is called **homosynchronous** if for all local p-states $x, y \in X \colon \mathcal{D}_p(x) = \mathcal{D}_p(y)$.

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  - for every run ρ of $\mathfrak{T}$ and every event $e \in \rho$, if $\Lambda(e) = \pi$ and $p \in \mathrm{dom}(e)$ then $\mathrm{ds}(e) = P \Leftrightarrow \mathcal{D}_p(\pi_{|p}) = P$

A set $X \subseteq X_p$ of local p-states is called **homosynchronous** if for all local p-states $x, y \in X \colon \mathcal{D}_p(x) = \mathcal{D}_p(y)$. For an infinite run ρ, processes p refer to the homosynchronous, **maximal local infinity sets**

$$\lceil \mathrm{Inf}_p(\rho) \rceil = \begin{cases} \left\{ x \in X_p \;\middle|\; \begin{array}{l} \mathcal{D}_p(x) = \lceil \mathrm{ds}_p(\rho) \rceil \wedge \\ \exists^{\infty} e \in \rho : \Lambda(e)_{|p} = x \end{array} \right\} & p \in \mathrm{dom}(\mathrm{alphinf}(\theta)) \\ \left\{ x \in X_p \;\middle|\; \begin{array}{l} \exists e \in \rho : e = \max_p(\rho) \\ \wedge\ \Lambda(e)_{|p} = x \end{array} \right\} & \text{otherwise.} \end{cases}$$

deterministic, synchronization-aware Büchi automata

A **D-SABA** is a tuple $\mathfrak{A} = (\mathfrak{T}, \mathcal{D}, \mathcal{F})$, where $(\mathfrak{T}, \mathcal{D})$ is an SATS, and the acceptance table $\mathcal{F} = \{(Q_1, F_1), \ldots (Q_k, F_k)\}$ is such that

- $Q_i \subseteq \mathcal{P}$, and
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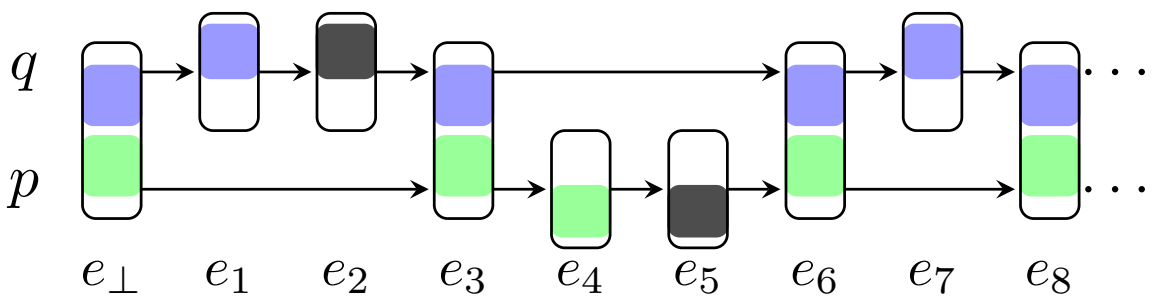

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Theorem (det. Büchi recognizable)

A language $\Theta \subseteq \mathbb{R}(\Sigma, I)$ is recognized by a D-SABA iff Θ is a deterministic trace language, i.e. iff Θ can be expressed as a finite union $\bigcup_i \lim_{A_i}(T_i)$ for regular trace languages $T_i \subseteq \mathbb{M}(\Sigma, I)$ and $A_i \subseteq \Sigma$.

deterministic, synchronization-aware Muller automata

A **D-SAMA** is a tuple $\mathfrak{A} = (\mathfrak{T}, \mathcal{D}, \mathcal{F})$, where $(\mathfrak{T}, \mathcal{D})$ is an SATS, and the acceptance table $\mathcal{F} = \{F_1, \dots F_k\}$ is such that $F_i = (F_i^p)_{p \in \mathcal{P}}$ are tuples of homosynchronous sets F_i^p.

deterministic, synchronization-aware Muller automata

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Theorem (ω-regular)

Any language $\Theta \subseteq \mathbb{R}(\Sigma, I)$ of infinite traces is recognized by a D-SAMA if and only if Θ is recognized by a DAMA.

closure under supersets

For tuples $F_1 = (F_1^p)_{p\in\mathcal{P}}, F_2 = (F_2^p)_{p\in\mathcal{P}}$ of homosynchronous sets, we say that F_1 **is a superset of** F_2 denoted $F_1 \supseteq F_2$ if for each $p \in \mathcal{P}, F_1^p \supseteq F_2^p$.

closure under supersets

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Table $\mathcal{F}$ is **closed under supersets** if $\big((F \in \mathcal{F}) \wedge (F' \supseteq F)\big) \Rightarrow (F' \in \mathcal{F})$

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Table $\mathcal{F}$ is **closed under supersets** if $\big((F \in \mathcal{F}) \wedge (F' \supseteq F)\big) \Rightarrow (F' \in \mathcal{F})$

Table $\mathcal{F}$ is said to be **closed under supersets modulo finitary acceptance sets** if

(a) whenever a tuple $F \in \mathcal{F}$ does not contain any "finitary acceptance sets" and there exists a tuple $F' \supseteq F$, then $F' \in \mathcal{F}$; and

(b) whenever at tuple $F \in \mathcal{F}$ contains a "finitary acceptance set" F^p and there exists a tuple $F' \supseteq F$ with $F'^p = F^p$, then $F' \in \mathcal{F}$.

language characterization

Theorem (L. Landweber)

A language $L \subseteq \Sigma^\omega$ is DBA recognizable iff for each Muller automaton recognizing L the acceptance component is closed under supersets.

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Theorem (characterization à la Landweber)

A language Θ is recognized by a D-SABA if and only if there exists a D-SAMA $\mathfrak{A} = (\mathfrak{T}, \mathcal{D}, \mathcal{F})$ recognizing Θ whose acceptance table $\mathcal{F}$ is closed under supersets modulo finitary acceptance sets.

language characterization

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Theorem (ω-regular $= \mathrm{BC}$(det. Büchi recognizable))

For any language $\Theta \subseteq \mathbb{R}(\Sigma, I)$ of infinite traces, Θ is D-SAMA recognizable if and only if Θ can be expressed as a finite Boolean combination of D-SABA recognizable languages.

conclusion

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- Generalizing the study of ω-regular languages

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  - A D-SABA accepts a language Θ if and only if $\Theta = \bigcup_{i=1}^{n} \lim_{A_i}(T_i)$
  - A characterization of det. trace languages in terms of a sub-class of det. Muller automata
  - An ω-regular trace language can be expressed as a Boolean combination of D-SABA-recognizable languages

conclusion

- Generalizing the study of ω-regular languages
  If the independence relation $I = \emptyset$
  - A D-SABA accepts a language Θ if and only if $\Theta = \bigcup_{i=1}^{n} \lim_{A_i}(T_i)$
    A DBA accepts a language L if and only if $L = \bigcup_{i=1}^{n} \lim_{\Sigma}(K_i)$
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    A characterization of det. languages in terms of a sub-class of det. Muller automata, leading to a decision process
  - An ω-regular trace language can be expressed as a Boolean combination of D-SABA-recognizable languages
    An ω-regular language can be expressed as a Boolean combination of DBA-recognizable languages

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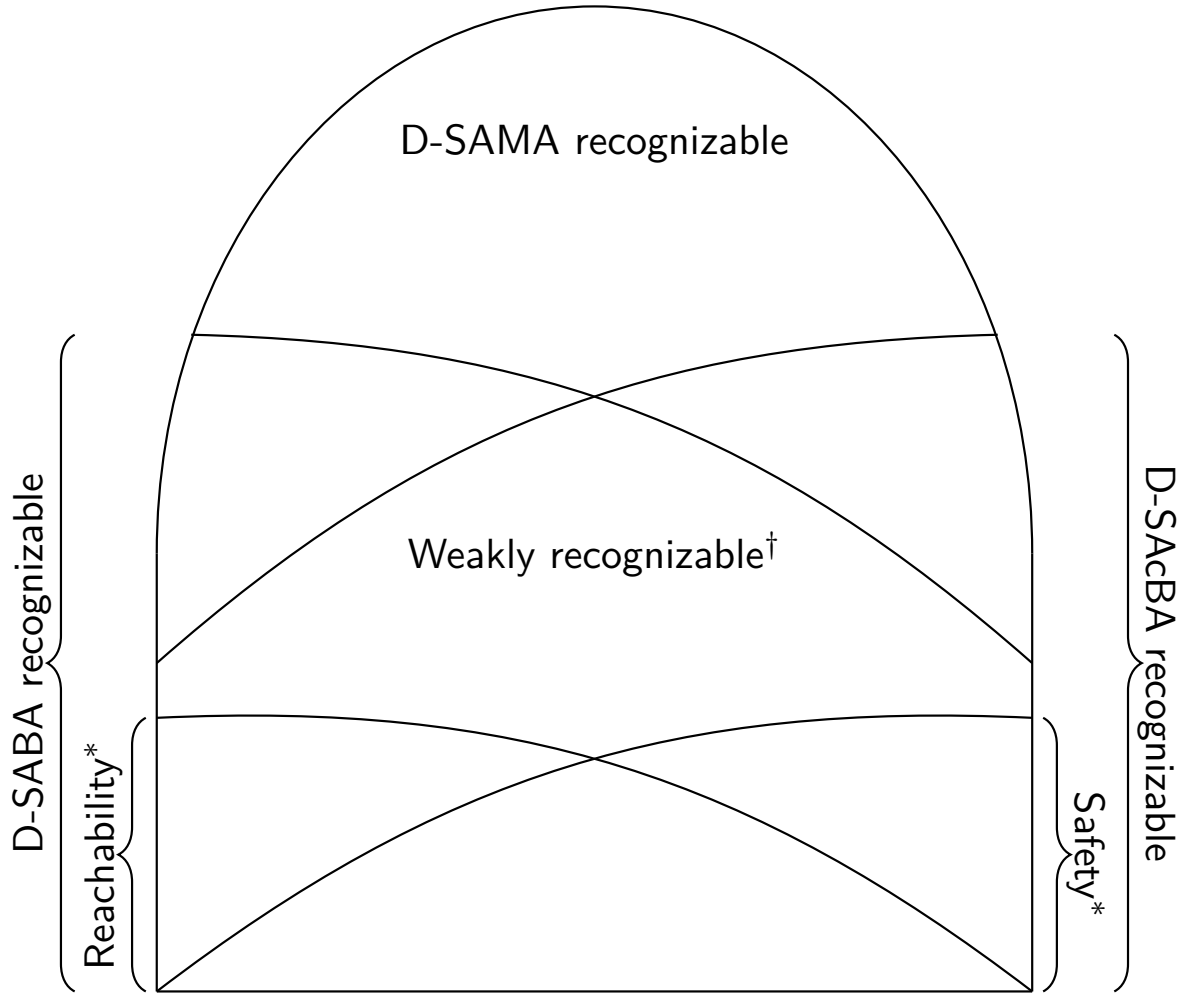
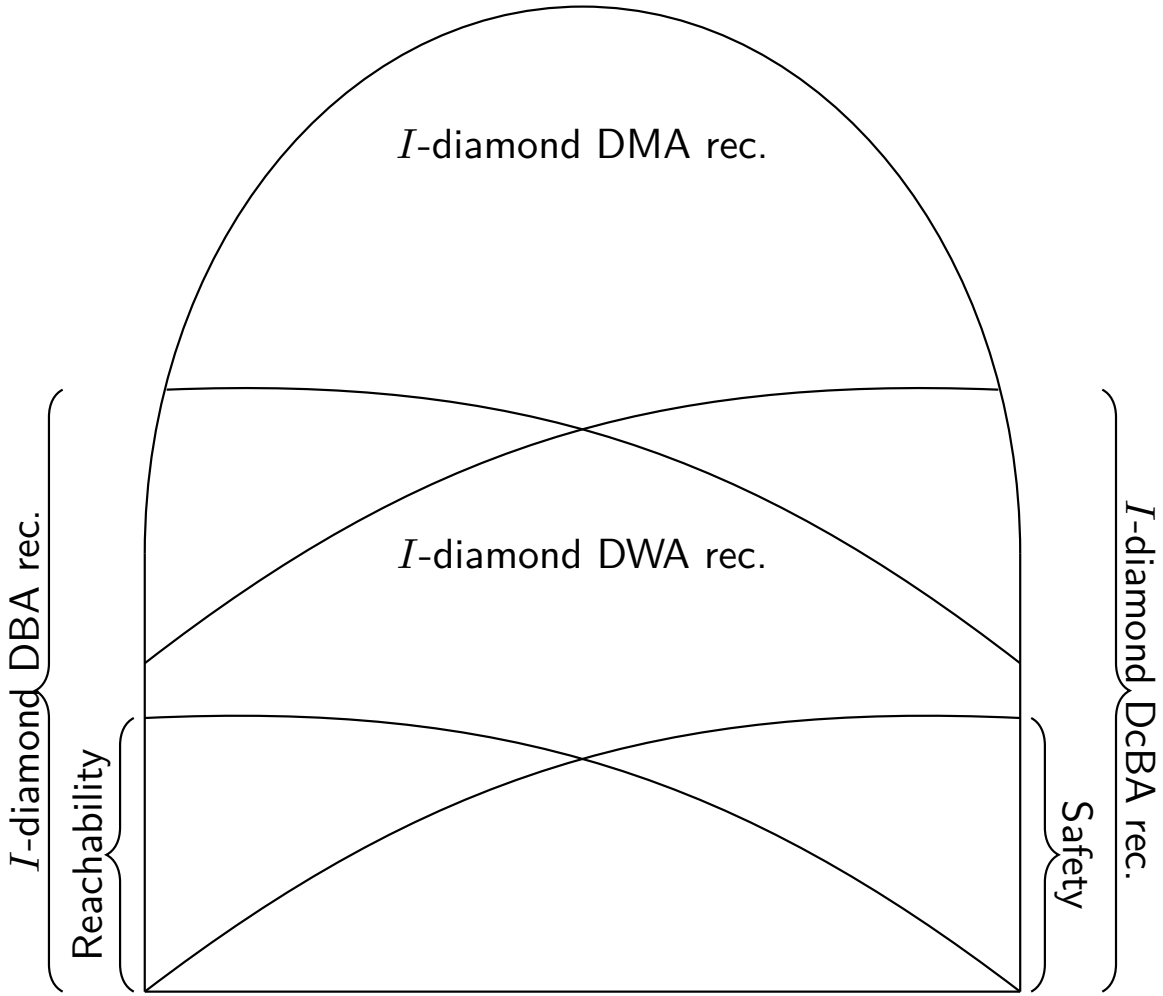
    An ω-regular language can be expressed as a Boolean combination of DBA-recognizable languages
- **In progress:** deterministic, synchronization-aware weak automata

a Borel-like hierarchy



Open: a characterization in terms of logic

a Borel-like hierarchy (part II)



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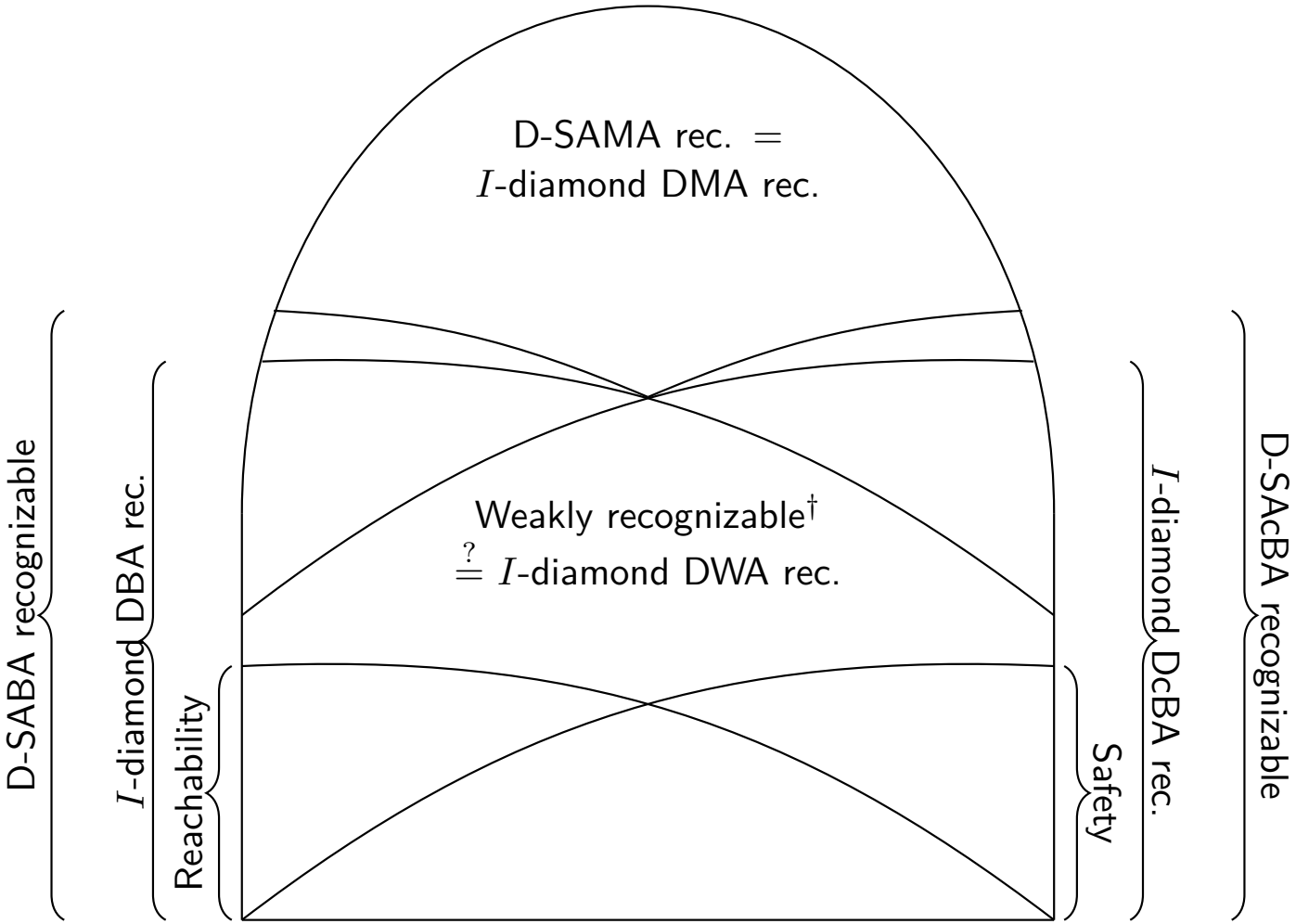

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