Symblicit algorithms for optimal strategy synthesis in monotonic Markov decision processes

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Overview (1/2)

Motivations:

- Markov decision processes with large state spaces
- Explicit enumeration exhausts the memory
- Symbolic representations like MTBDDs are useful
- No easy use of (MT)BDDs for solving linear systems

[1] R. Wimmer, B. Braitling, B. Becker, E. M. Hahn, P. Crouzen, H. Hermanns, A. Dhama, and O. E. Theel. Symblicit calculation of long-run averages for concurrent probabilistic systems. In *QEST*, pages 27-36. IEEE Computer Society, 2010.

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Recent contributions of [WBB+10][1]:

- **Symblicit** algorithm
  - Mixes **symb**olic and exp**licit** data structures
- Expected mean-payoff in Markov decision processes
- Using (MT)BDDs

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Overview (2/2)

Our contributions:

- New structure of **pseudo-antichain** (extension of antichains)
  $\Rightarrow$ Pseudo-antichain based symblicit algorithms
- **Monotonic** Markov decision processes
- **Two quantitative settings**:
  - Expected mean-payoff
  - Stochastic shortest path (focus of this talk)
- **Two applications**:
  - Automated planning
  - LTL synthesis

Full paper available on ArXiv: abs/1402.1076

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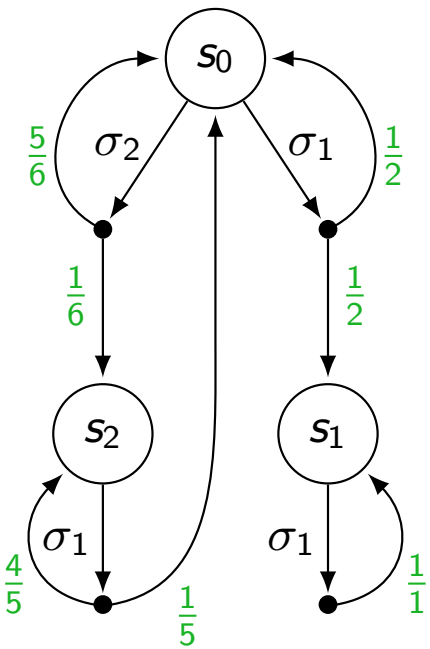
Markov decision processes (MDPs)



- $M = (S, \Sigma, \mathcal{P})$ where:
  - S is a finite set of *states*
  - Σ is a finite set of *actions*
  - $\mathcal{P} : S \times \Sigma \to \mathit{Dist}(S)$ is a *stochastic transition function*

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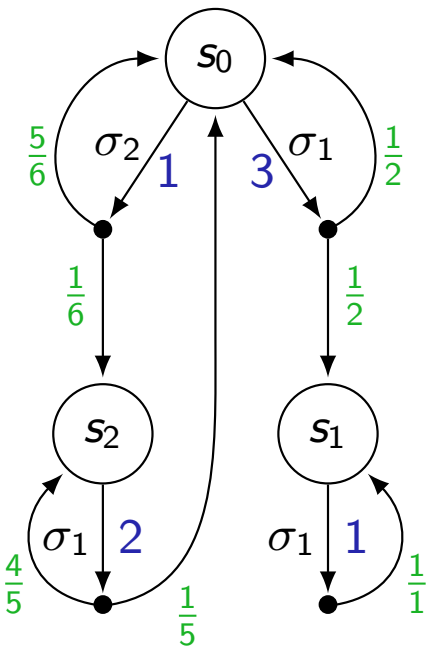

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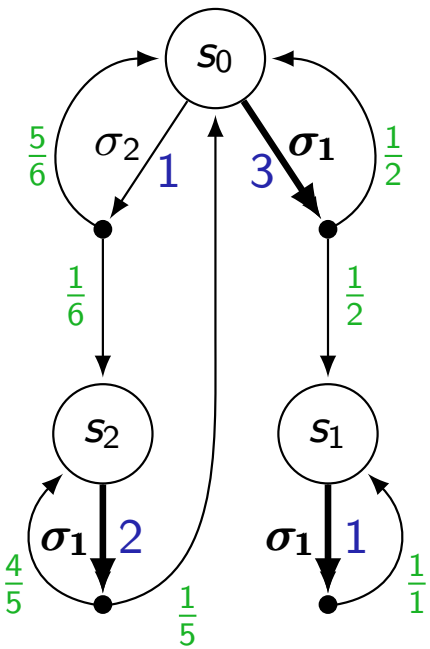

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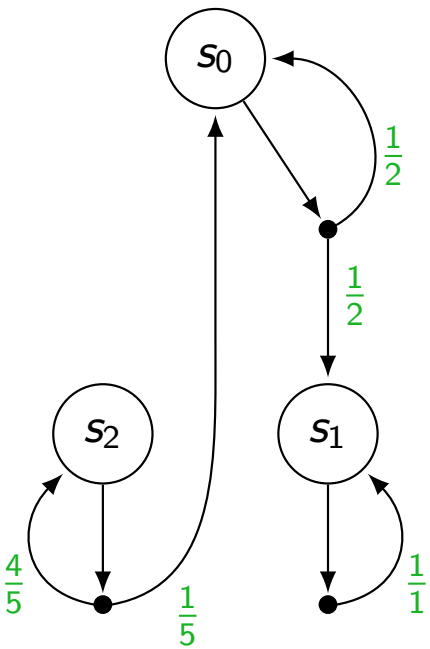
- (Memoryless) strategy $\lambda : S \to \Sigma$

Markov chains (MCs)



- MDP $(S, \Sigma, \mathcal{P})$ with $\mathcal{P} : S \times \Sigma \to \mathit{Dist}(S)$
  $+$ strategy $\lambda : S \to \Sigma$
  $\Rightarrow$ *induced* MC $(S, \mathcal{P}_\lambda)$ with $\mathcal{P}_\lambda : S \to \mathit{Dist}(S)$

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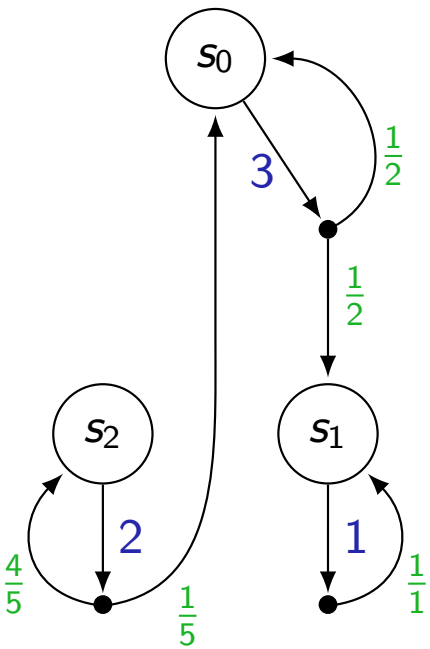

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- Cost function $c : S \times \Sigma \to \mathbb{R}_{>0}$

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Expected truncated sum

- Let $M_\lambda = (S, \mathcal{P}_\lambda)$ with cost function c_λ
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- $\mathbb{E}_\lambda^{\mathrm{TS}_G}(s) = \sum_\rho \mathcal{P}_\lambda(\rho)\mathrm{TS}_G(\rho)$, with $\rho = s_0 s_1 \dots s_n$ s.t.
  $s_0 = s, s_n \in G$ and $s_0, \dots, s_{n-1} \notin G$

Stochastic shortest path (SSP)

- Let $M = (S, \Sigma, \mathcal{P})$ with cost function c
- Let $G \subseteq S$ be a set of *goal states*
- λ^* is *optimal* if $\mathbb{E}_{\lambda^*}^{\mathsf{TS}_G}(s) = \inf_{\lambda \in \Lambda} \mathbb{E}_{\lambda}^{\mathsf{TS}_G}(s)$

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- λ^* is *optimal* if $\mathbb{E}^{\mathsf{TS}_G}_{\lambda^*}(s) = \inf_{\lambda \in \Lambda} \mathbb{E}^{\mathsf{TS}_G}_{\lambda}(s)$
- SSP problem: compute an optimal strategy λ^*
- Complexity and strategies [BT96]:
  - Polynomial time via linear programming
  - Memoryless optimal strategies exist

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Ingredients

- Strategy iteration algorithm [How60, BT96]
  - Generates a sequence of **monotonically improving strategies**
  - 2 phases:
    - strategy evaluation by solving a linear system
    - strategy improvement at each state
  - Stops as soon as no more improvement can be made
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- Bisimulation lumping [LS91, Buc94, KS60]
  - Applies to MCs
  - Gathers states which behave equivalently
  - Produces a **bisimulation quotient** (hopefully) smaller
  - Interested in the *largest* bisimulation $\sim_L$

Symblicit algorithm

- Mix of symbolic and explicit data structures

Algo 1 Symblicit(MDP $M^{\mathcal{S}}$, Cost function $c^{\mathcal{S}}$, Goal states $G^{\mathcal{S}}$)

$n := 0, \lambda_n^{\mathcal{S}} :=$ InitialStrategy$(M^{\mathcal{S}}, G^{\mathcal{S}})$
repeat
  $(M_{\lambda_n}^{\mathcal{S}}, c_{\lambda_n}^{\mathcal{S}}) :=$ InducedMCAndCost$(M^{\mathcal{S}}, c^{\mathcal{S}}, \lambda_n^{\mathcal{S}})$
  $(M_{\lambda_n,\sim_L}^{\mathcal{S}}, c_{\lambda_n,\sim_L}^{\mathcal{S}}) :=$ Lump$(M_{\lambda_n}^{\mathcal{S}}, c_{\lambda_n}^{\mathcal{S}})$
  $(M_{\lambda_n,\sim_L}, c_{\lambda_n,\sim_L}) :=$ Explicit$(M_{\lambda_n,\sim_L}^{\mathcal{S}}, c_{\lambda_n,\sim_L}^{\mathcal{S}})$
  $v_n :=$ SolveLinearSystem$(M_{\lambda_n,\sim_L}, c_{\lambda_n,\sim_L})$
  $v_n^{\mathcal{S}} :=$ Symbolic(v_n)
  $\lambda_{n+1}^{\mathcal{S}} :=$ ImproveStrategy$(M^{\mathcal{S}}, \lambda_n^{\mathcal{S}}, v_n^{\mathcal{S}})$
  $n := n + 1$
until $\lambda_n^{\mathcal{S}} = \lambda_{n-1}^{\mathcal{S}}$
return $(\lambda_{n-1}^{\mathcal{S}}, v_{n-1}^{\mathcal{S}})$

Key: $\mathcal{S}$ in superscript denotes symbolic representations

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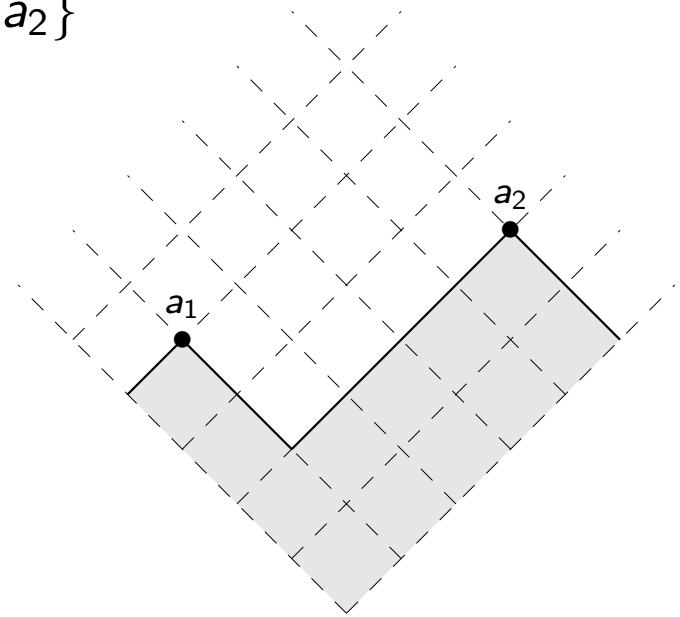
Antichains

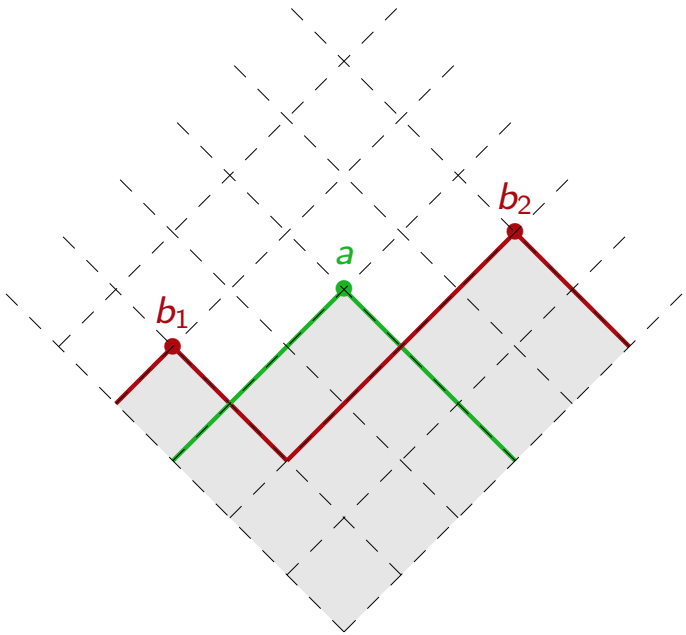
- Let $(S, \preceq)$ be a semilattice with greatest lower bound $\sqcap$
- A set $\alpha \subseteq S$ is an **antichain** if $\forall s, s' \in \alpha$, $s \not\preceq s'$ and $s' \not\preceq s$
- The *closure* of α is $\downarrow\alpha = \{s \in S \mid \exists a \in \alpha, s \preceq a\}$
- Example: $\alpha = \{a_1, a_2\}$

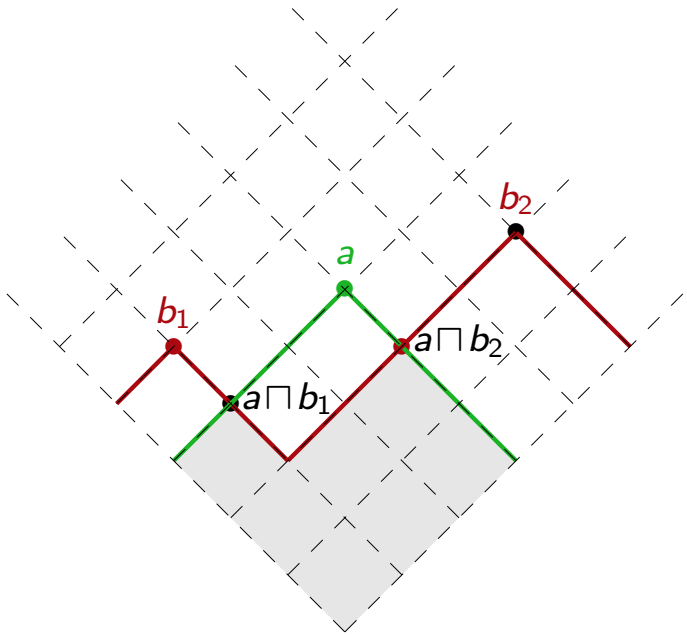


- **Canonical representations** of **closed sets** L by their maximal elements $\lceil L \rceil$ (*unique*)

Operations on antichains

- Given two antichains $\alpha, \beta \subseteq S$
- $\downarrow\alpha \cup \downarrow\beta$ and $\downarrow\alpha \cap \downarrow\beta$ are **closed**:

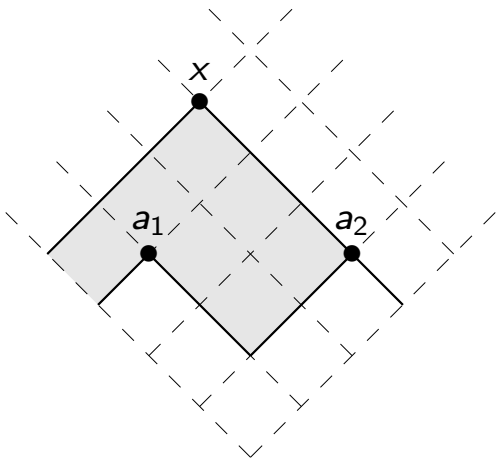




- But $\downarrow\alpha \setminus \downarrow\beta$ is **not** necessarily **closed**

Pseudo-elements

- Let $(S, \preceq)$ be a semilattice with greatest lower bound $\sqcap$
- A **pseudo-element** is a pair (x, α) where $x \in S$ and $\alpha \subseteq S$ is an antichain such that $x \notin \downarrow\alpha$
- The *pseudo-closure* of (x, α) is $\updownarrow(x, \alpha) = \{s \in S \mid s \preceq x \text{ and } s \notin \downarrow\alpha\}$
  $= \downarrow\{x\} \setminus \downarrow\alpha$
- Example: (x, α) with $\alpha = \{a_1, a_2\}$



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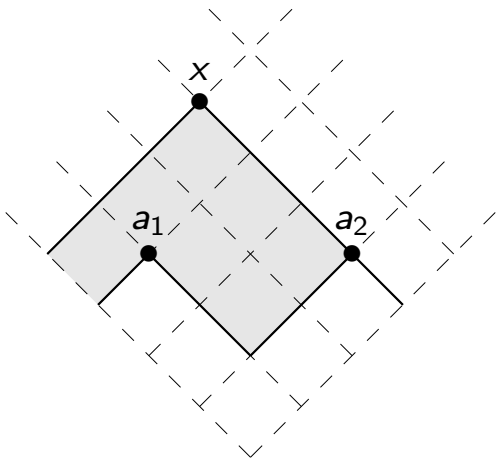

- (x, α) is in **canonical form** if $\forall a \in \alpha, a \preceq x$ (*unique*)

Pseudo-antichains

- A **pseudo-antichain** A is a set $\{(x_i, \alpha_i) \mid i \in I\}$ of pseudo-elements
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  - $\forall i \in I, (x_i, \alpha_i)$ is in canonical form, and
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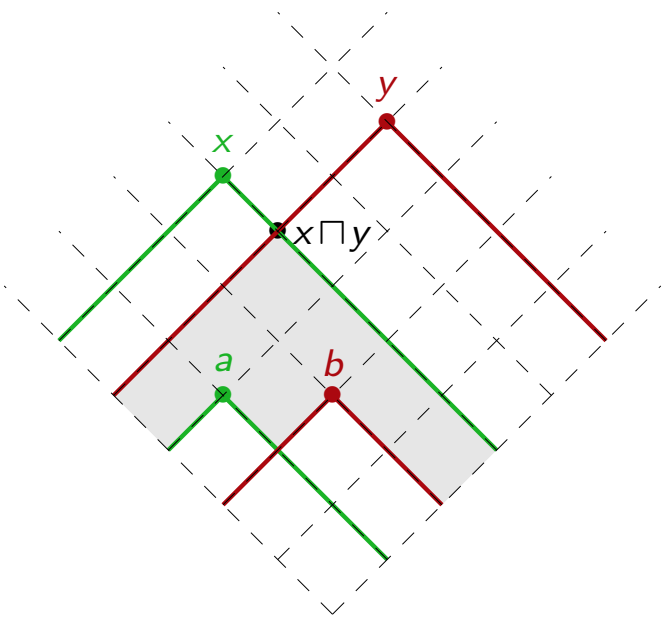
  - A does not contain unnecessary pseudo-elements
- Remark: **any** set can be PA-represented

Operations

- Efficient computations of pseudo-closures of pseudo-antichains w.r.t. the union, intersection and difference
- Example (for the intersection):
  $\updownarrow(x, \{a\}) \cap \updownarrow(y, \{b\}) = \updownarrow(x \sqcap y, \{a, b\})$



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New definition of MDPs

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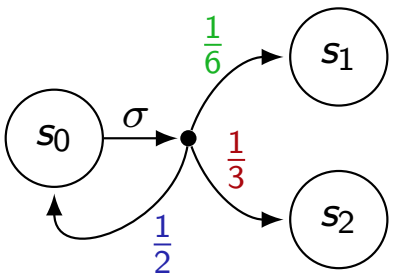
- MDP $M = (S, \Sigma, T, \mathcal{E}, \mathcal{D})$ where:
  - T set of stochastic actions s.t. $\Sigma \cap T = \emptyset$
  - $\mathcal{E} : S \times \Sigma \to \mathcal{F}_{\text{tot}}(T, S)$ **successor function**
  - $\mathcal{D} : S \times \Sigma \to \mathit{Dist}(T)$ **stochastic function**

$M = (S, \Sigma, \mathcal{P})$



$\mathcal{P}(s_0, \sigma)(s_0) = \frac{1}{2}$
$\mathcal{P}(s_0, \sigma)(s_1) = \frac{1}{6}$
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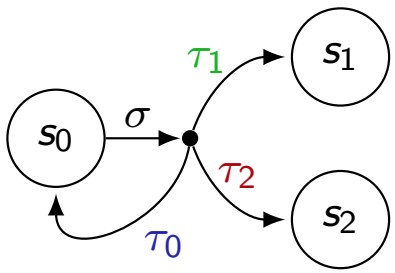

$\mathcal{E}(s_0, \sigma)(\tau_0) = s_0$
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Monotonic MDPs

- A *monotonic* MDP is an MDP $M_{\preceq} = (S, \Sigma, T, \mathcal{E}, \mathcal{D})$ s.t.:
  - S is equipped with a partial order $\preceq$ s.t. $(S, \preceq)$ is a semilattice

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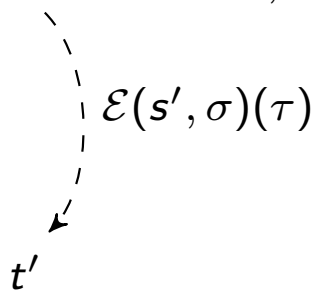
$$s \preceq s' \quad \forall\, \sigma \in \Sigma, \tau \in T$$



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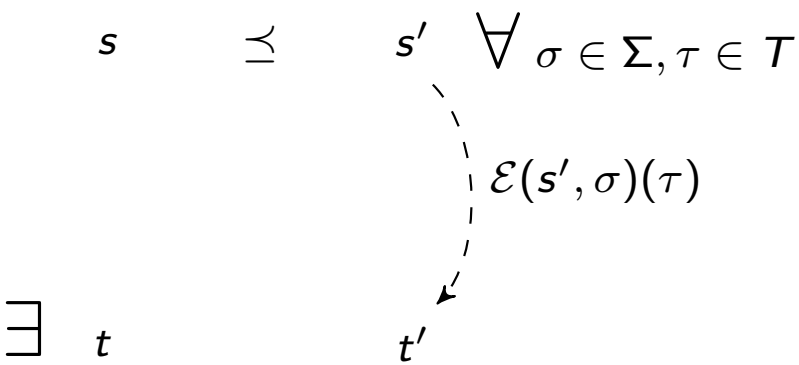

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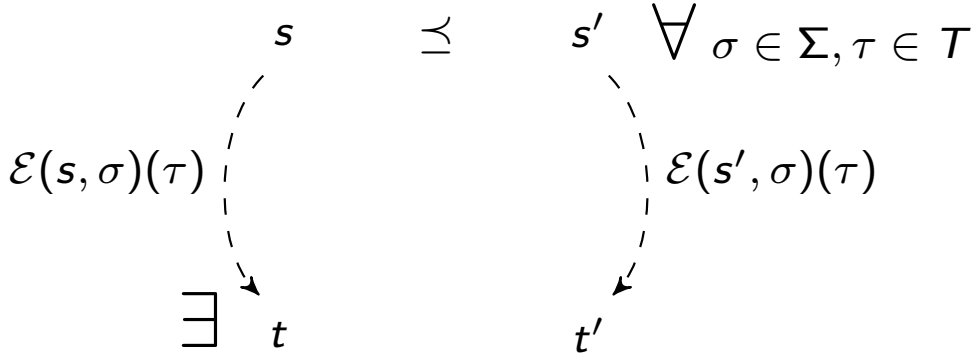

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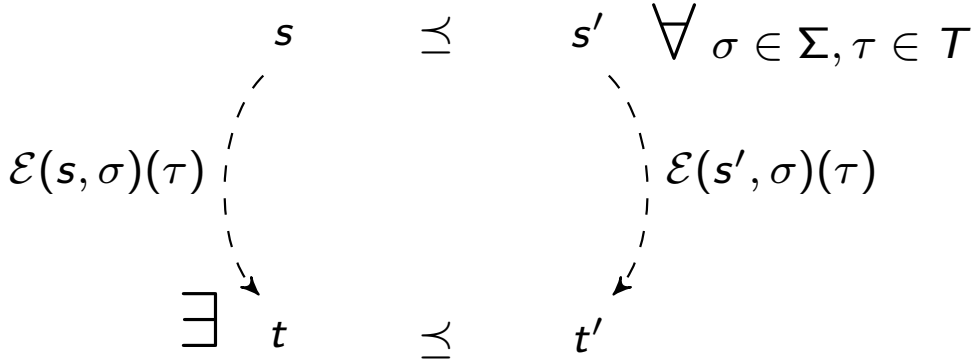

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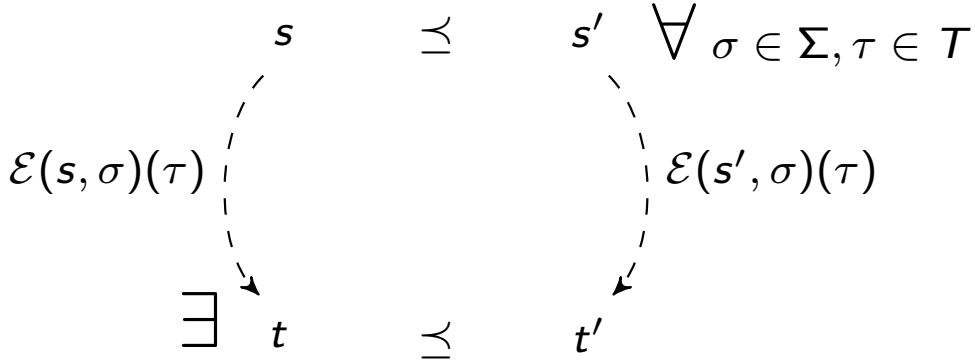
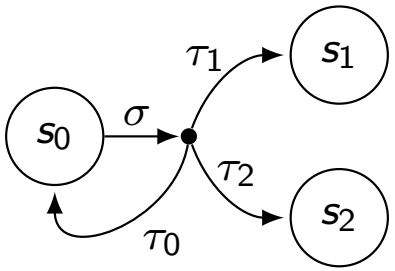


- Remark:
  - All MDPs can be seen monotonic
  - Interested in MDPs built on state spaces *already* equipped with a **natural partial order**

$\mathrm{Pre}_{\sigma,\tau}$ operator

Given $L \subseteq S, \sigma \in \Sigma$ and $\tau \in T$, let

$$\mathrm{Pre}_{\sigma,\tau}(L) = \{s \in S \mid \mathcal{E}(s,\sigma)(\tau) \in L\}$$



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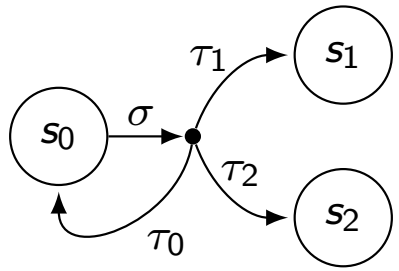

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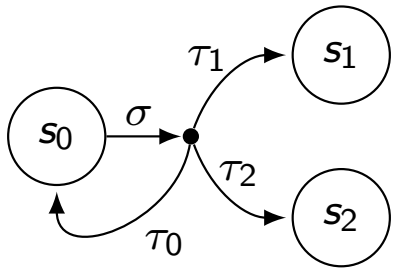

Lemma. For all closed set $L \subseteq S$, $\mathrm{Pre}_{\sigma,\tau}(L)$ is closed

Proposition. Let $A = \{(x_i, \alpha_i) \mid i \in I\}$ be a pseudo-antichain. Then,

- $\mathrm{Pre}_{\sigma,\tau}(\updownarrow A) = \bigcup_{i \in I} \mathrm{Pre}_{\sigma,\tau}(\updownarrow(x_i, \alpha_i))$

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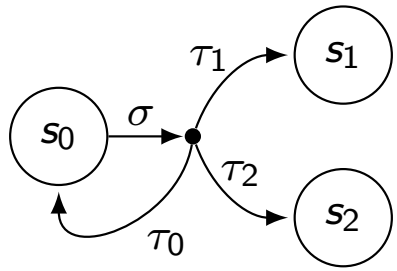

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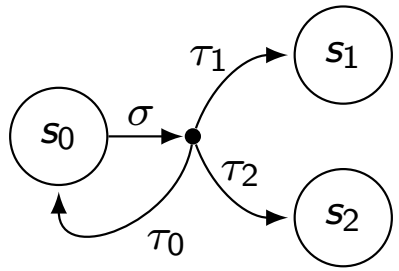

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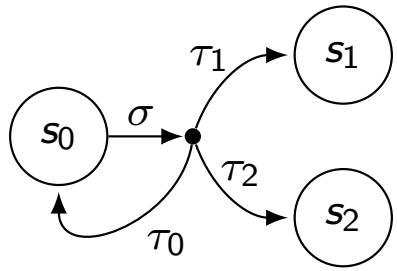

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$\Rightarrow$ The computation of $\mathrm{Pre}_{\sigma,\tau}(\updownarrow A)$ does not treat the whole $\updownarrow A$

Symbolic representations

- Representations of equivalence relations
  - Example: $s \sim_\lambda s'$ iff $\lambda(s) = \lambda(s')$
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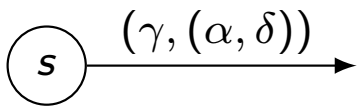

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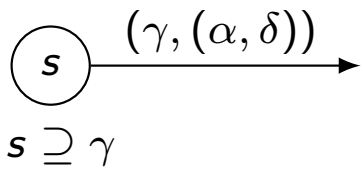

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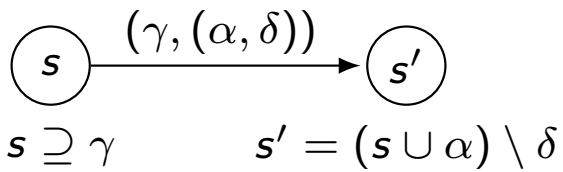

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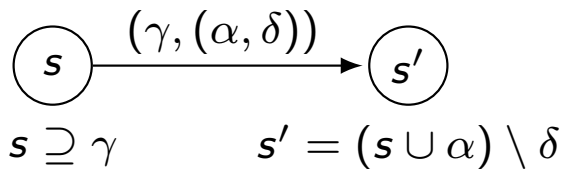

Planning from STRIPS [FN72]

- Find a sequence of operators leading from the initial state I to a goal state $s \supseteq M$

Stochastic STRIPS

- Extension of STRIPS with **stochastic aspects** [BL00]
- Operator $o = (\gamma, \pi) \in O$ s.t.
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$\pi(\alpha_1, \delta_1) = \frac{2}{5}$
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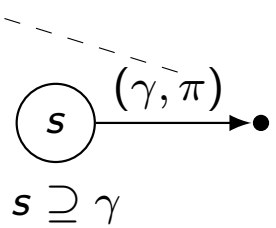

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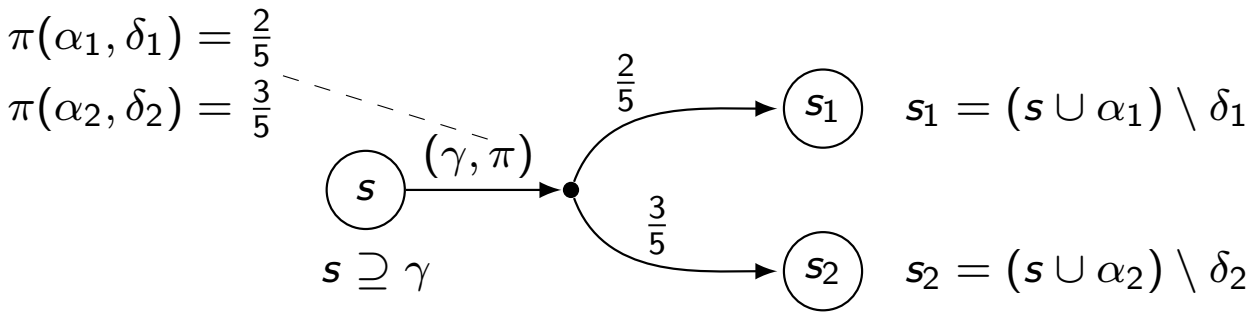

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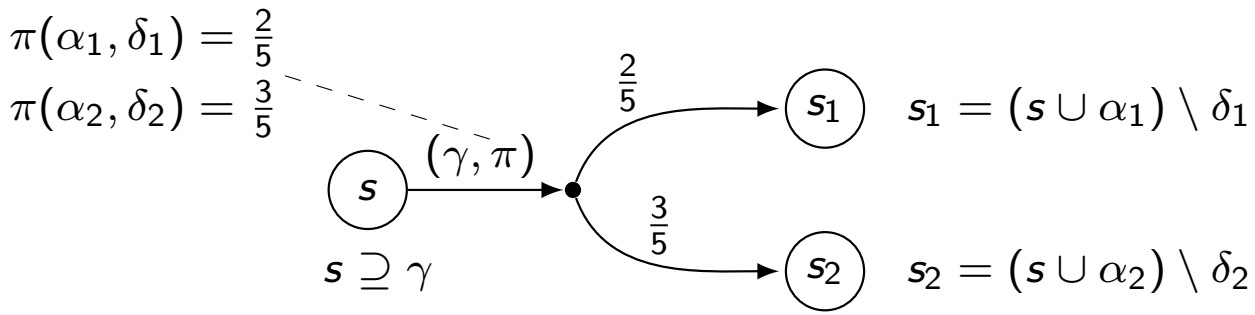

- $\rightsquigarrow$ Markov decision process

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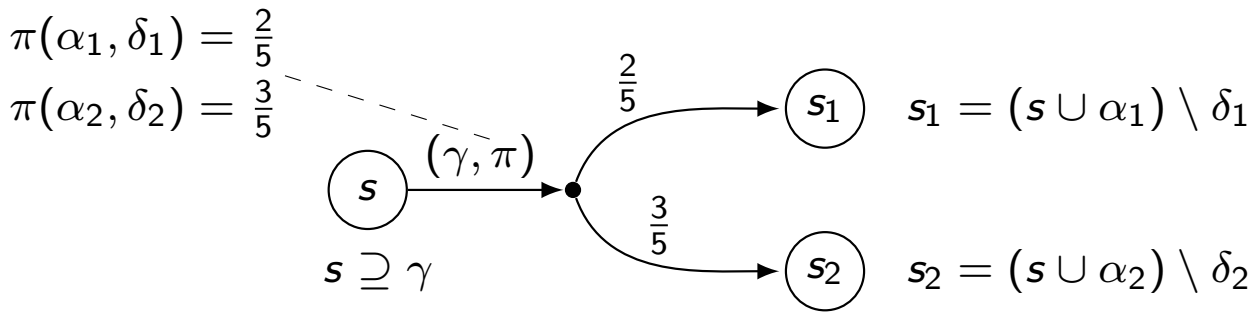

- $\rightsquigarrow$ Markov decision process

- Cost function $C : O \to \mathbb{R}_{>0}$
- Planning from stochastic STRIPS
  - Minimize the truncated sum up to a state $s \supseteq M$ from I
  - $\rightsquigarrow$ Stochastic shortest path problem

Symblicit algorithm (1/2)

- M_S is **monotonic**
  - $(S, \supseteq)$ is a semilattice with greatest lower bound $\sqcap = \cup$
  - $\supseteq$ is compatible with $\mathcal{E}$

$$s \qquad \supseteq \qquad s'$$

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$$s \quad \supseteq \quad s' \quad \forall\, \sigma \in \Sigma, \tau = (\alpha, \delta) \in T$$

$$\mathcal{E}(s', \sigma)(\tau)$$

$$(s' \cup \alpha) \setminus \delta = t'$$

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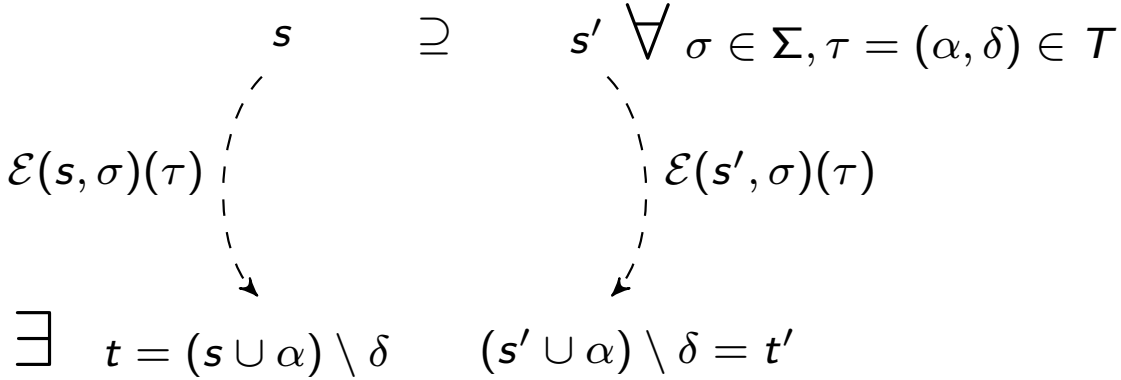

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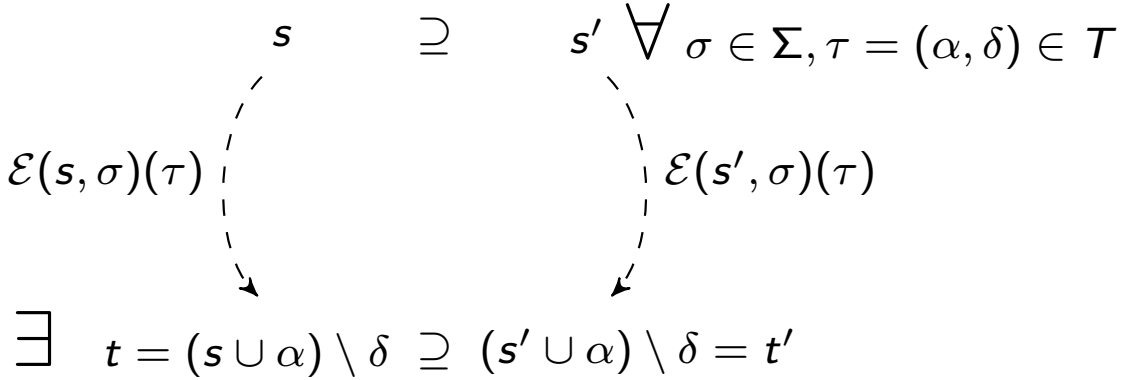

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- Assumption 1. There exists an algorithm taking any state $x \in S$ in input and returning $\lceil \mathrm{Pre}_{\sigma,\tau}(\downarrow\{x\}) \rceil$:

Algo 2 $\mathrm{Pre}(x, \sigma = (\gamma, \pi), \tau = (\alpha, \delta))$

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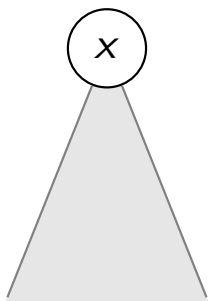

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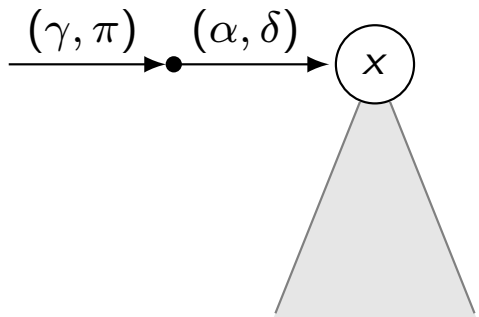

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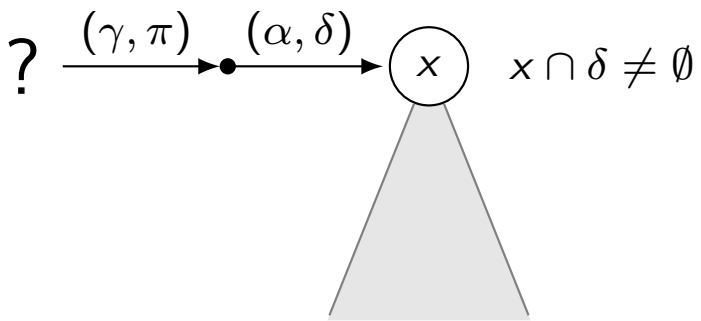

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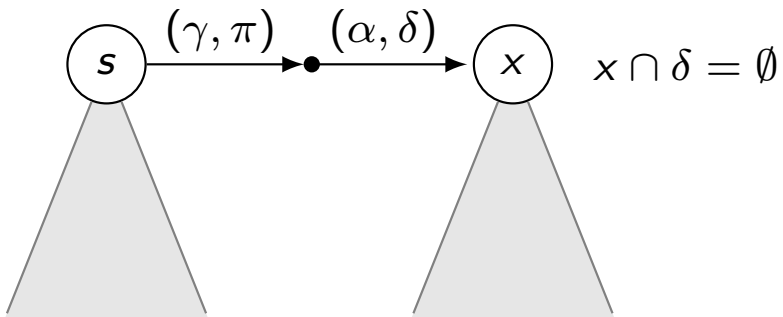

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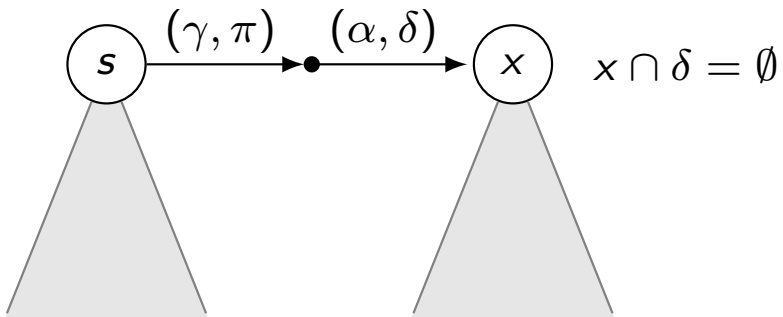

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  - $\mathcal{E}(s, \sigma)(\tau) = (s \cup \alpha) \setminus \delta$
  - $\mathcal{D}(s, \sigma)(\tau) = \pi(\tau)$

Experimental results

	example	$\mathbb{E}_\lambda^{\mathsf{TS}_G}$	$\lvert M_S \rvert$	PA #it	PA $\lvert \sim_L \rvert$	PA time	PA mem	Explicit time	Explicit mem
Monkey	(3, 2)	35.75	4096	4	23	0.2	16.0	60.6	1626
	(3, 3)	35.75	65536	5	43	1.6	17.3		> 4000
	(3, 4)	35.75	1048576	6	57	17.8	21.7		> 4000
	(3, 5)	36.00	16777216	7	88	272.1	37.5		> 4000
	(5, 2)	35.75	65536	4	31	0.5	16.6	20316.2	2343
	(5, 3)	35.75	4194304	5	56	8.2	19.5		> 4000
	(5, 4)	35.75	268435456	6	97	196.8	31.3		> 4000
	(5, 5)	36.00	17179869184	7	152	7098.4	81.3		> 4000
Moats and castles	(2, 5)	32.22	4096	3	49	1.8	17.3	133.7	1202
	(2, 6)	32.22	16384	3	66	11.7	19.3	2966.8	1706
	(3, 3)	59.00	4096	3	84	15.3	20.2	149.6	1205
	(3, 4)	52.00	32768	3	219	150.8	30.7	14660.7	1611
	(3, 5)	48.33	262144	3	357	740.2	49.1		> 4000
	(3, 6)	48.33	2097152	3	595	11597.7	145.8		> 4000
	(4, 2)	96.89	4096	3	132	43.7	26.5	173.6	1211
	(4, 3)	78.67	65536	3	464	1594.5	82.2		> 4000

Expected mean-payoff with LTL synthesis

Comparison with an MTBDD based symblicit algorithm [VE13]

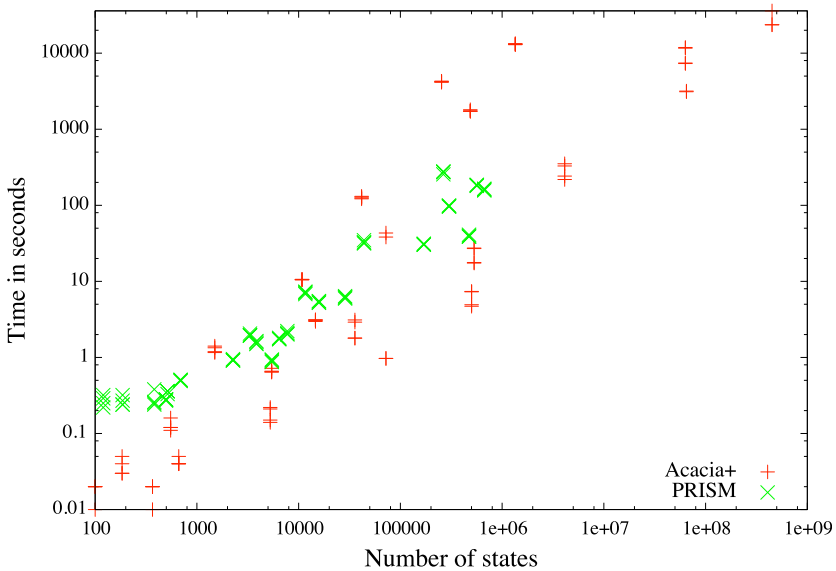

Figure : Execution time

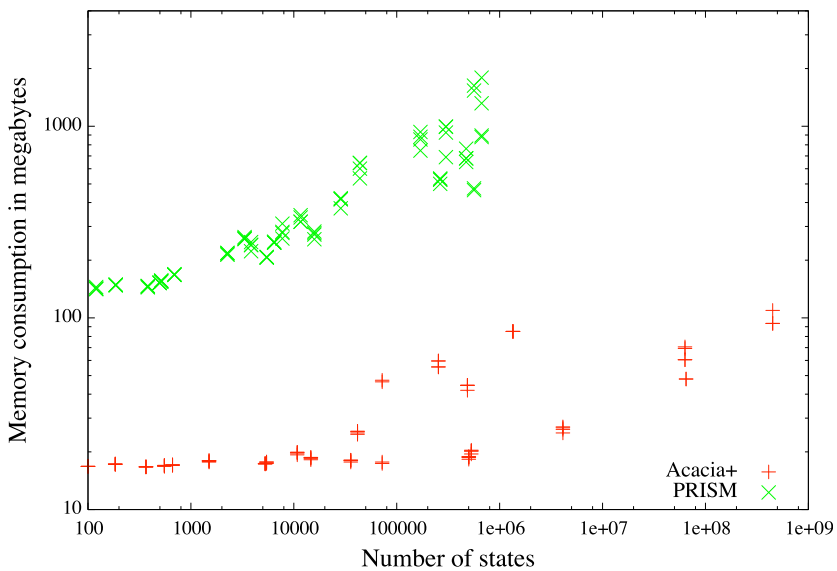

Figure : Memory consumption

$\Rightarrow$ Monotonic MDPs are better handled by pseudo-antichains

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Conclusion and future work

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- New data structure of **pseudo-antichains**
- Symblicit algorithms in monotonic MDPs with a **natural partial order**
- Expected mean-payoff and stochastic shortest path
- Promising experimental results

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- New data structure of **pseudo-antichains**
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Future work:

- Implementation of a MTBDD based symblicit algorithm for the stochastic shortest path
- Apply pseudo-antichains in other contexts (e.g. model-checking or synthesis of non-stochastic models)

Thank you!

Questions?

References I

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LTL synthesis with mean-payoff objectives

- Synthesis from LTL specifications with MP objectives [BBFR13]
- Reduction to a 2-player safety game $(\mathcal{G}, \alpha)$ with a partial order
- Winning strategies represented by $\mathsf{Win}_1(\mathcal{G}, \alpha)$

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- Goal: compute the **best strategy against a stochastic opponent**
- Replace the second player by a probability distribution in $\mathsf{Win}_1(\mathcal{G}, \alpha)$
- $\Rightarrow$ Monotonic MDP that satisfies Assumptions 1 and 2
- Symblicit algorithm for the expected mean-payoff problem
- Implementation in Acacia+